DAMPING CONTROL NOISE REDUCTION in KAGRA

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Abstract: Damping control noise is a task to be adressed to achieve the target sinsitivity of KAGRA in low frequency (~ 60 Hz). This poster shows the method using optimal estimator as one of the solutions to this problem.

01. Introduction

- KAGRA's mirror is suspended by the pendulum. e.g.) cryogenic payload for main mirrors **(Fig1)**
- Damping control noise of the suspension is a big problem in low frequency region (~ 60 Hz) when we aim for our target sensitivity **(Fig2)**.
- This noise comes from our reflective photosensor, whose mech. & noise level are shown in **(Fig3)**.

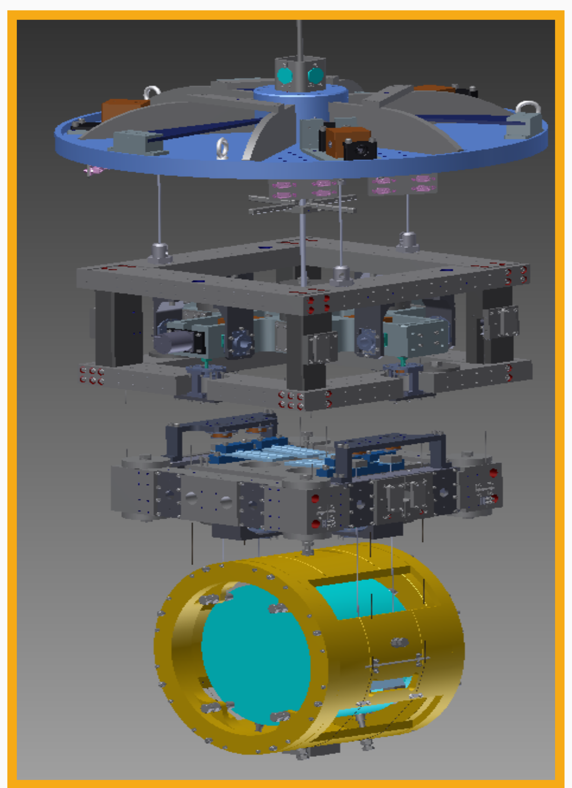

Fig1. Cryogenic payload

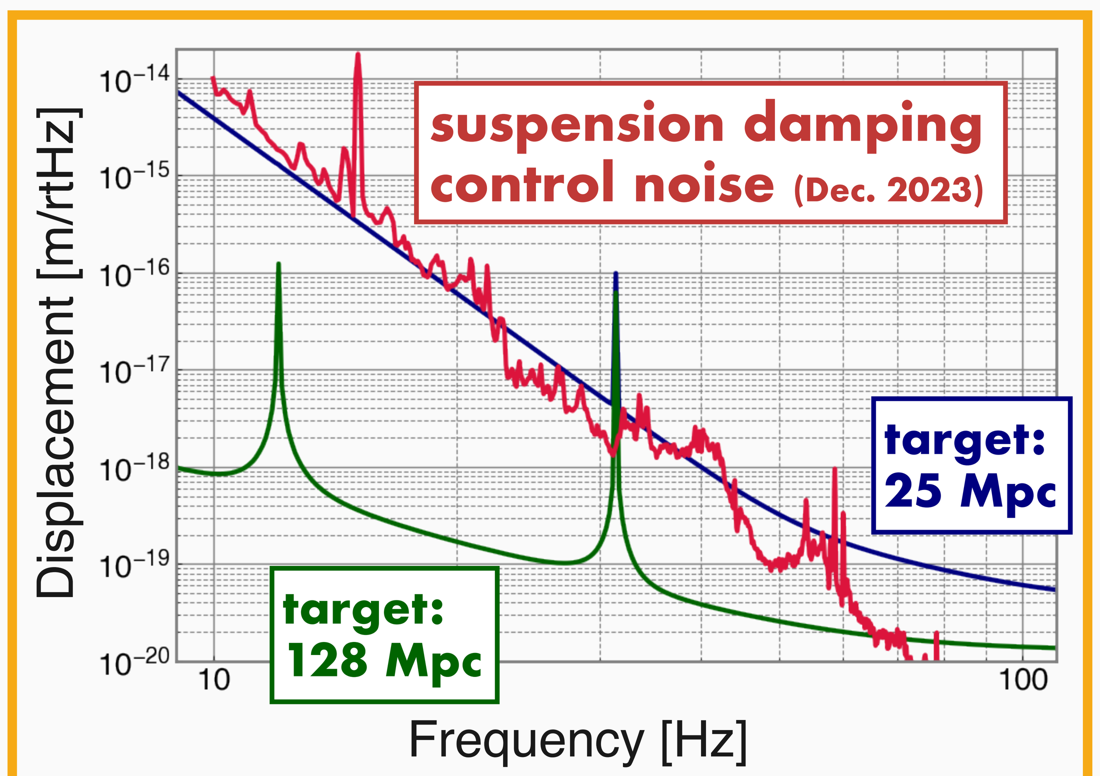

Fig2. Target sensitivity (BNS)[1] & Damping control noise

02. Solutions

- A modal model is applied **to control each mode**, but since it is difficult to measure all DoFs to know the "full state", state estimation is used **(Fig4)**.

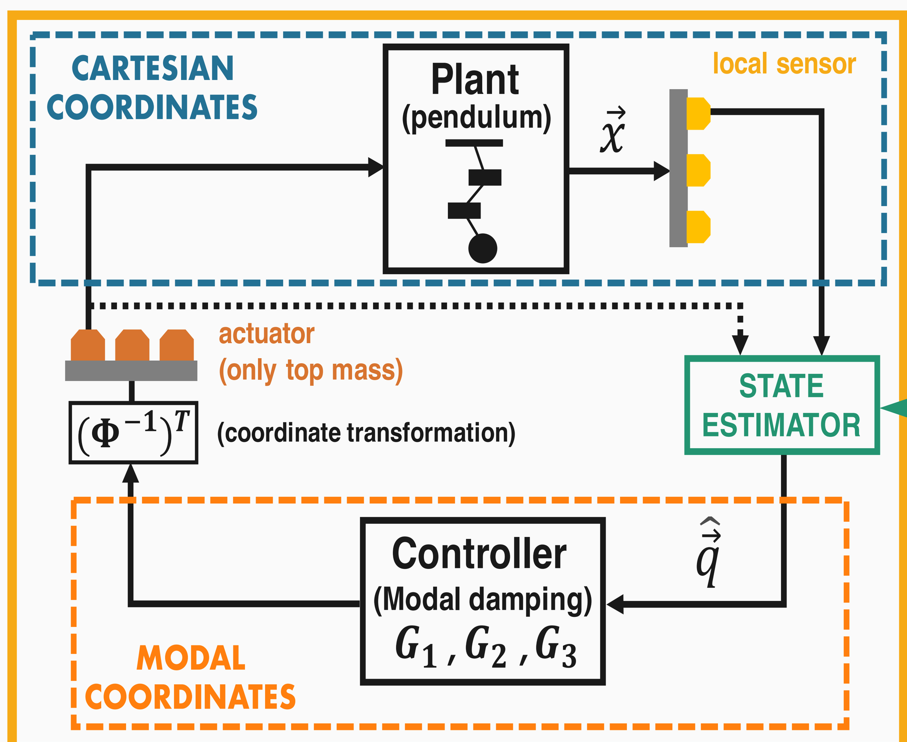

Fig4. Control loop with state estimator

State estimator

$$\dot{\hat{q}} = A_m\hat{q} + B_m u + L_m(y - C_m\hat{q}), \quad \hat{y} = C_m\hat{q}$$

q: modal state u: control inputs

y: sensor output L: estimator feedback gain

▷ **System**

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

A: state matrix
B: input matrix
C: output matrix

▷ **Coordinate transformation**

$x = \Phi q$ (Φ is calculated from stiffness & mass matrix)

GOAL

design of L_m [how the estimation error evolves]

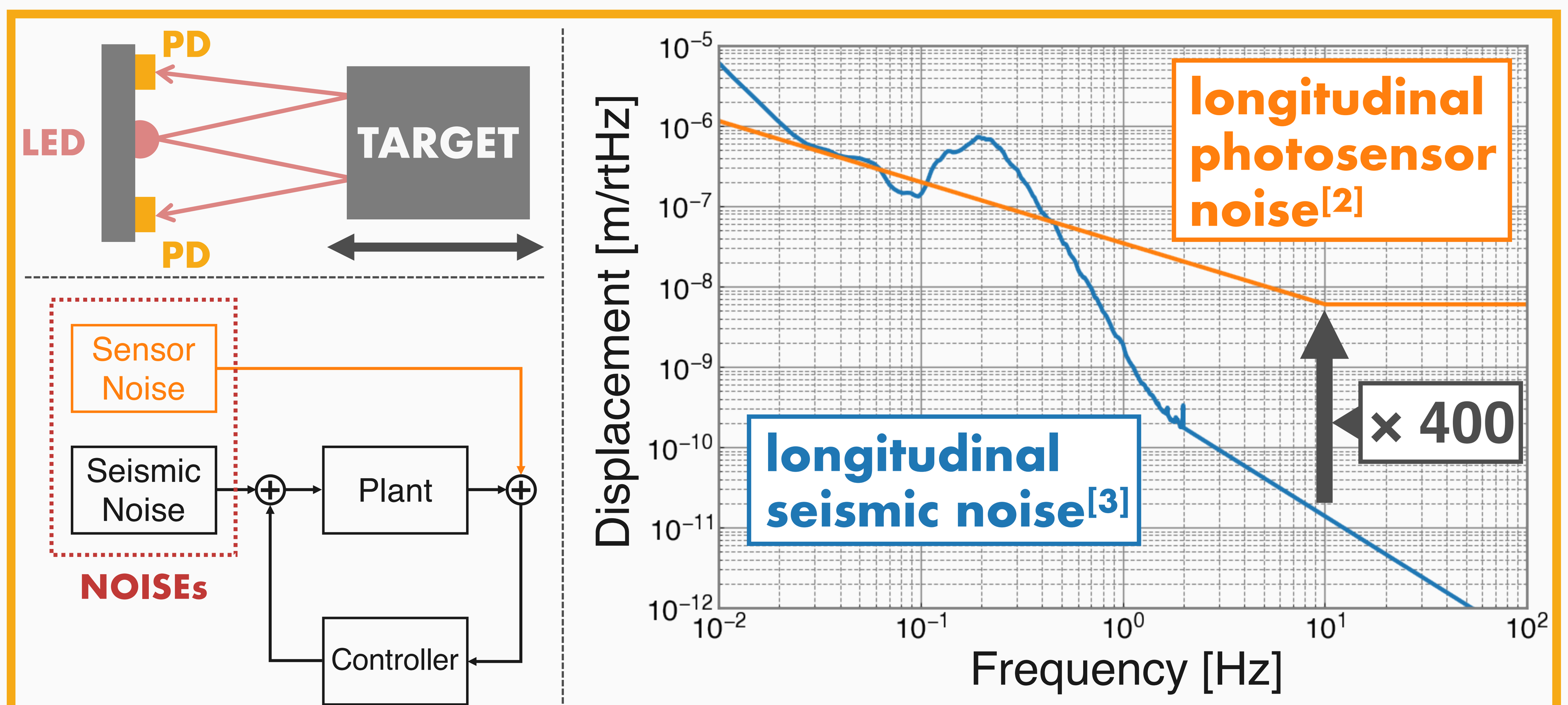

Fig3. Reflective photsensor, Control loop, Noise inputs for longitudinal (beam axis) loop

How to design a estimator which reduces the sensor noise transmission

LQR : automatically find an **optimal** feedback = **minimize the noise transmitted to TM**

▷ **Cost function**

$$J = \int_0^\infty (x^T Q x + u^T R u)\,dt$$

we want to select weights in freq. domain [4]
[Q: weight on estimation error R: weight on sensor output]

⬇ applying Parseval's theorem

$$J = \frac{1}{2}\int_{-\infty}^{\infty} (\bar{x}(-j\omega)^T\bar{x}(j\omega) + \bar{u}(-j\omega)^T\bar{u}(j\omega))\,d\omega$$

⬇ applying Parseval's theorem

$$J = \int_0^\infty (x^T H_x^T H_x x + 2x^T H_x^T G_x \chi + \chi^T G_x^T G_x \chi + \mu^T G_u^T G_u \mu + 2\mu^T G_u^T H_u u + u^T H_u^T H_u u)\,dt$$

$$= \int_0^\infty (z^T Q z + u^T R u + 2z^T N u)\,dt$$

$$Q = \begin{bmatrix} Q_m & 0 \\ 0 & G^T G \end{bmatrix}, \quad R = H^T H, \quad N = \begin{bmatrix} 0 \\ G_u^T H_u \end{bmatrix}$$

mode reconstruction cost: $\begin{bmatrix} H_x^T H_x & H_x^T G_x \\ G_x^T H_x & G_x^T G_x \end{bmatrix}$

Filtering function
$$\begin{cases} \dot{\chi} = E_x\chi + F_x x \\ \dot{\mu} = E_u\mu + F_u u \end{cases}$$

Filtered states
$$\begin{cases} \bar{x} = G_x\chi + H_x x \\ \bar{u} = G_u\mu + H_u u \end{cases}$$

Augmented state
$$z = [x\ \chi\ \mu]^T, \quad \dot{z} = \bar{A}z + \bar{B}u$$

$$\bar{A} = \begin{bmatrix} A_m^T & 0 \\ 0 & E \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} C_m^T \\ F \end{bmatrix}$$

» L_m is calculated by "`lqr(sys,Q,R,N)`" in MATLAB (solving ARE: $P\bar{A} + \bar{A}^T P - (P\bar{B} + N)R^{-1}(P\bar{B} + N)^T + Q = 0$)

03. Results

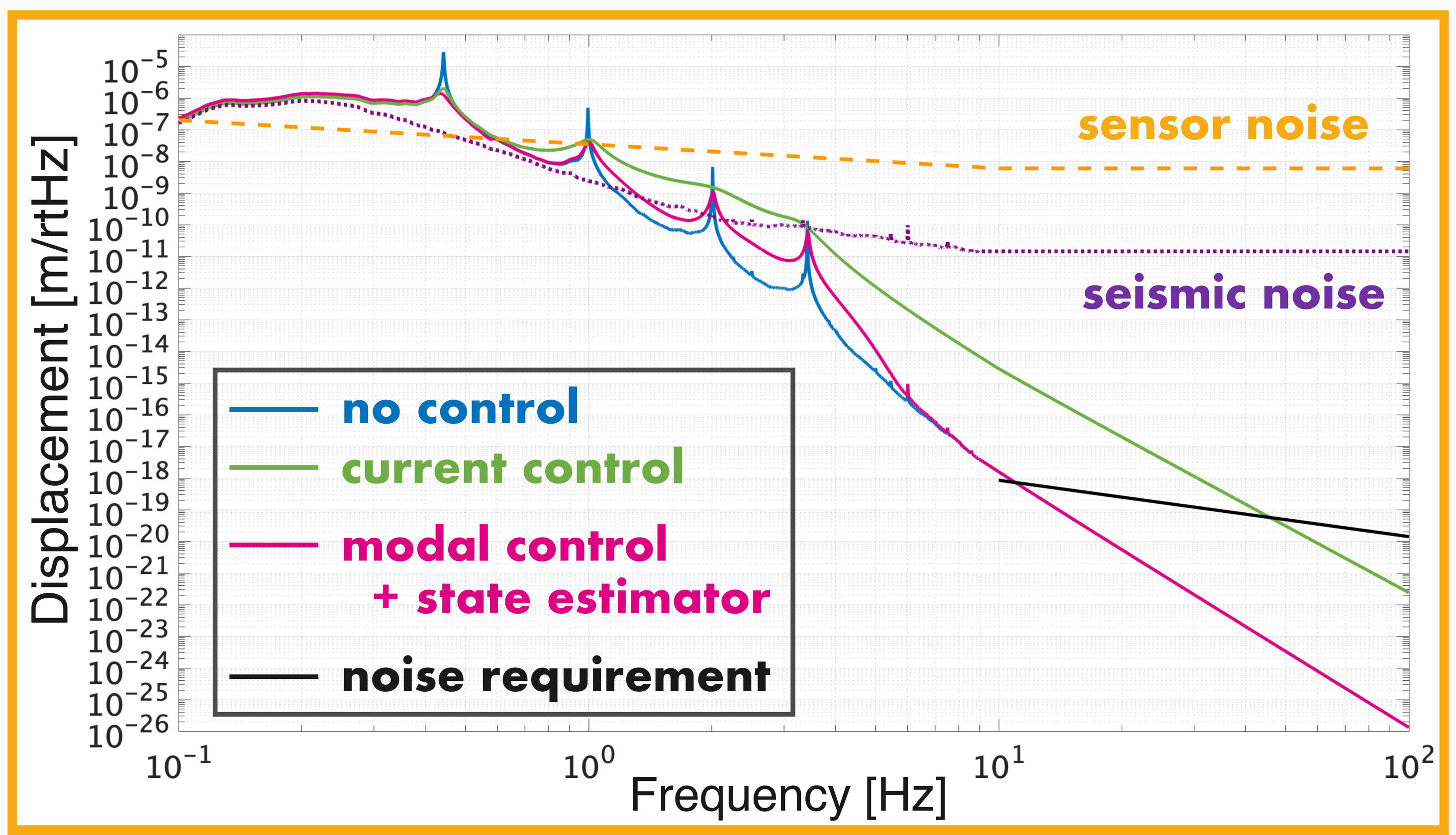

Fig5. TM noise response (Longitudinal)

04. Discussion & Outlook

- Simulation result is good, so **this is one candidate for the improvement**.
- Choice of the weight may be a problem.
- Sensor update should also be considered.

Reference [1] Y. Michimura, JGW-T2113624 [2] T. Ushiba, klog17939 [3] K. Miyo, JGW-T1910436 [4] N. Gupta, JGCD, 3(6), (1980)