LVK collaboration meeting Mar. 11 (2024)

KAGRA

Damping Control Noise Reduction for KAGRA Suspension

LIGO VIRGO KAGRA

KAGRA Suspensions

Problems

Solution (Future Works)

Masahide TAMAKI

the University of Tokyo / Institute for Cosmic Ray Research

AGENDA

1 KAGRA Suspensions

2 Problems

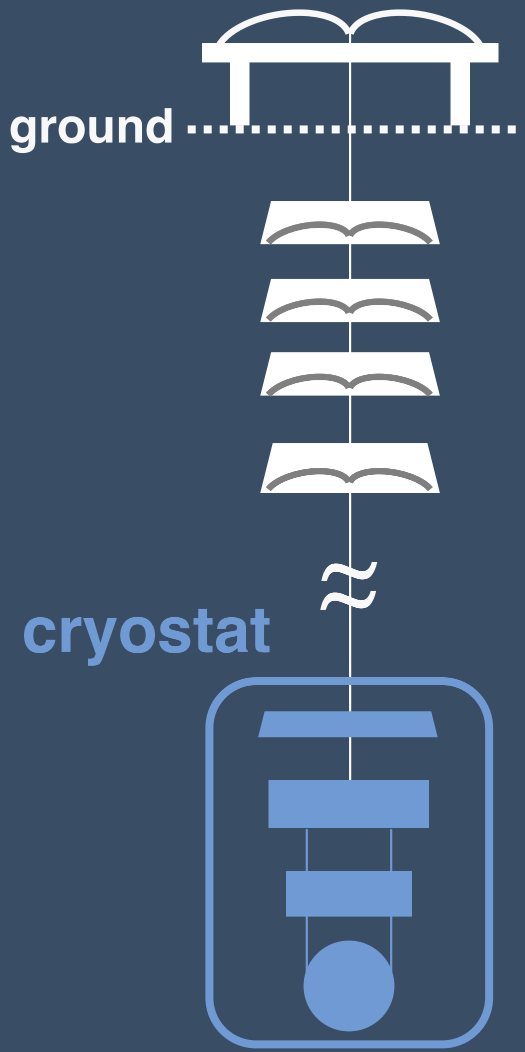
3 Solutions (Future Works)

KAGRA Suspensions

Type-A (9 stages · 13.5 m)

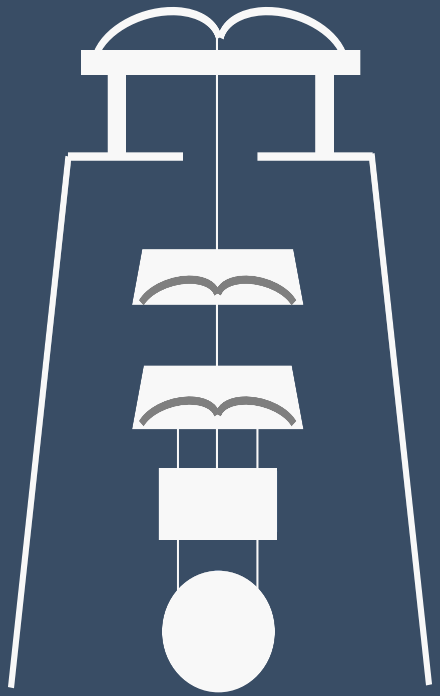
For main mirrors

Tower + Cryogenic Payload



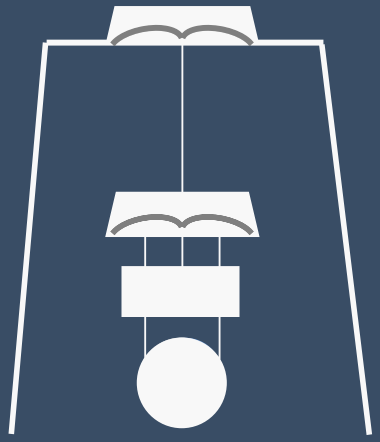
Type-B

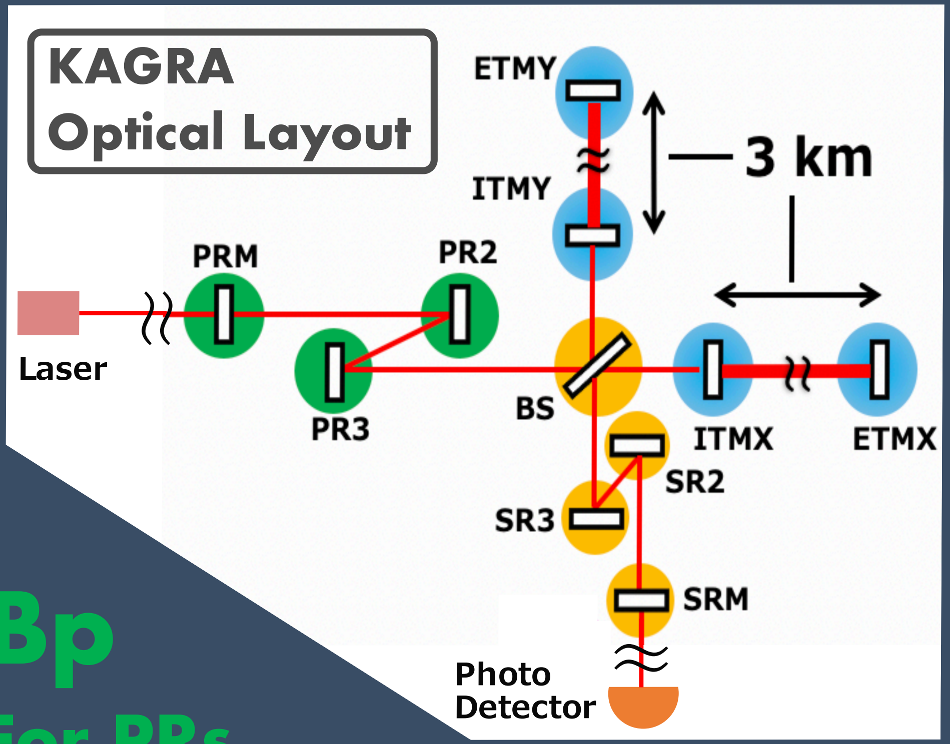
For BS & SRs



Type-Bp

For PRs

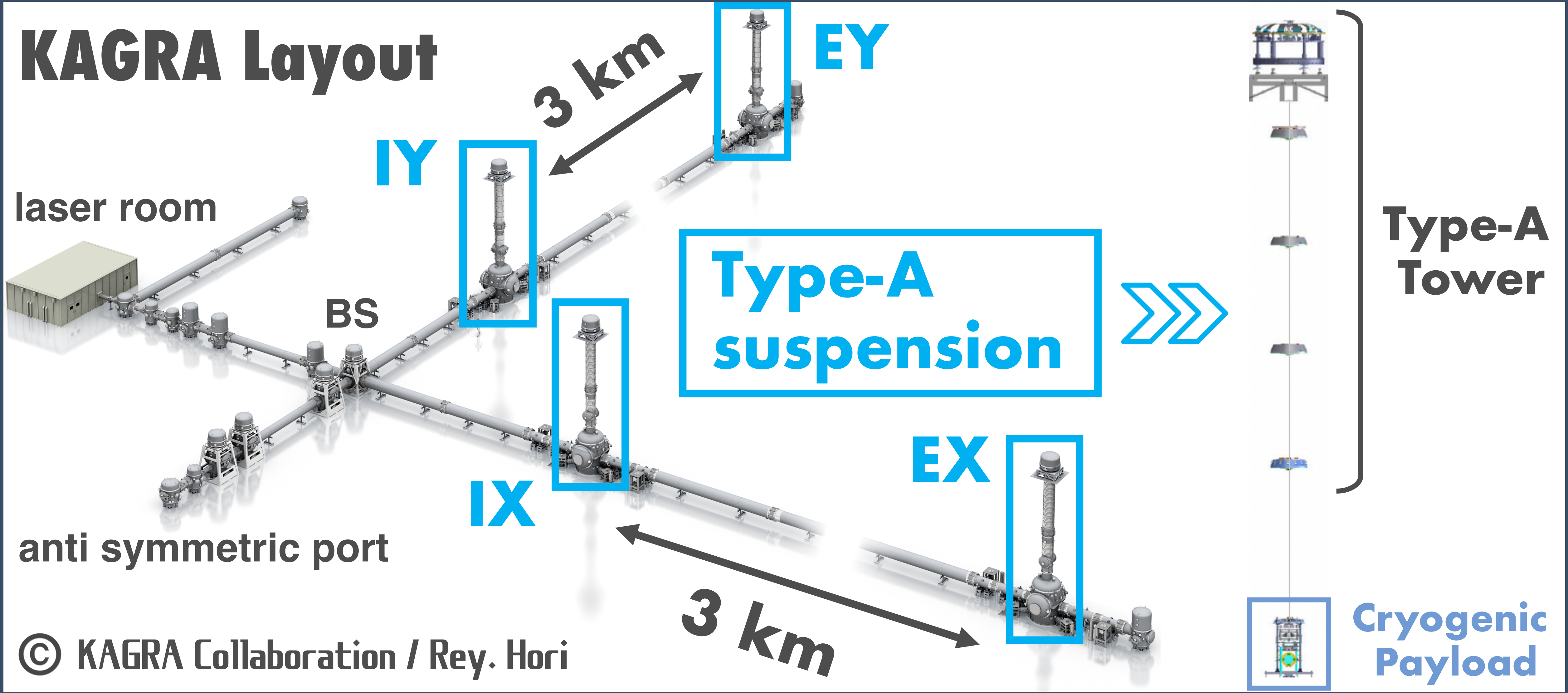




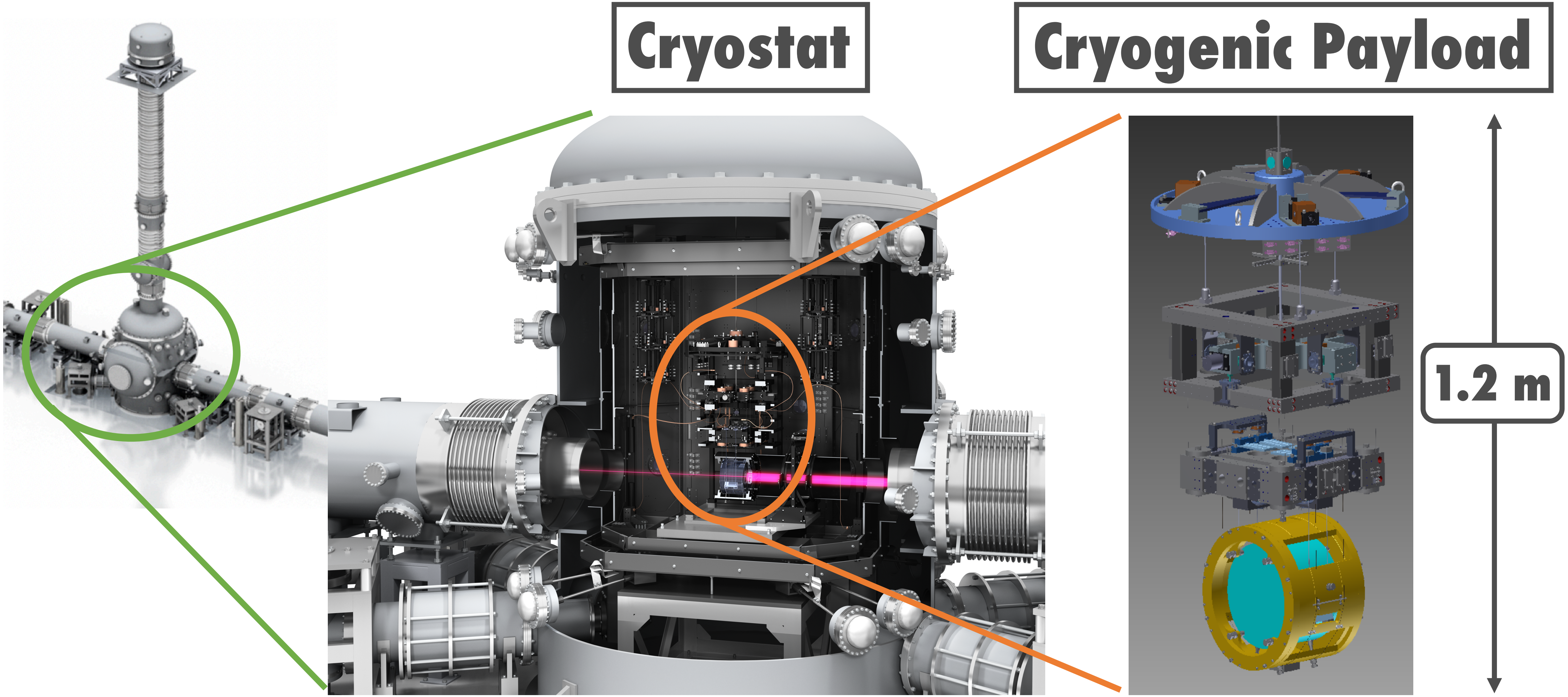
Although omitted in the figure, IMC/OMC are suspended by mini suspension called Type-C

Type-A Suspension

(for main sapphire mirror)

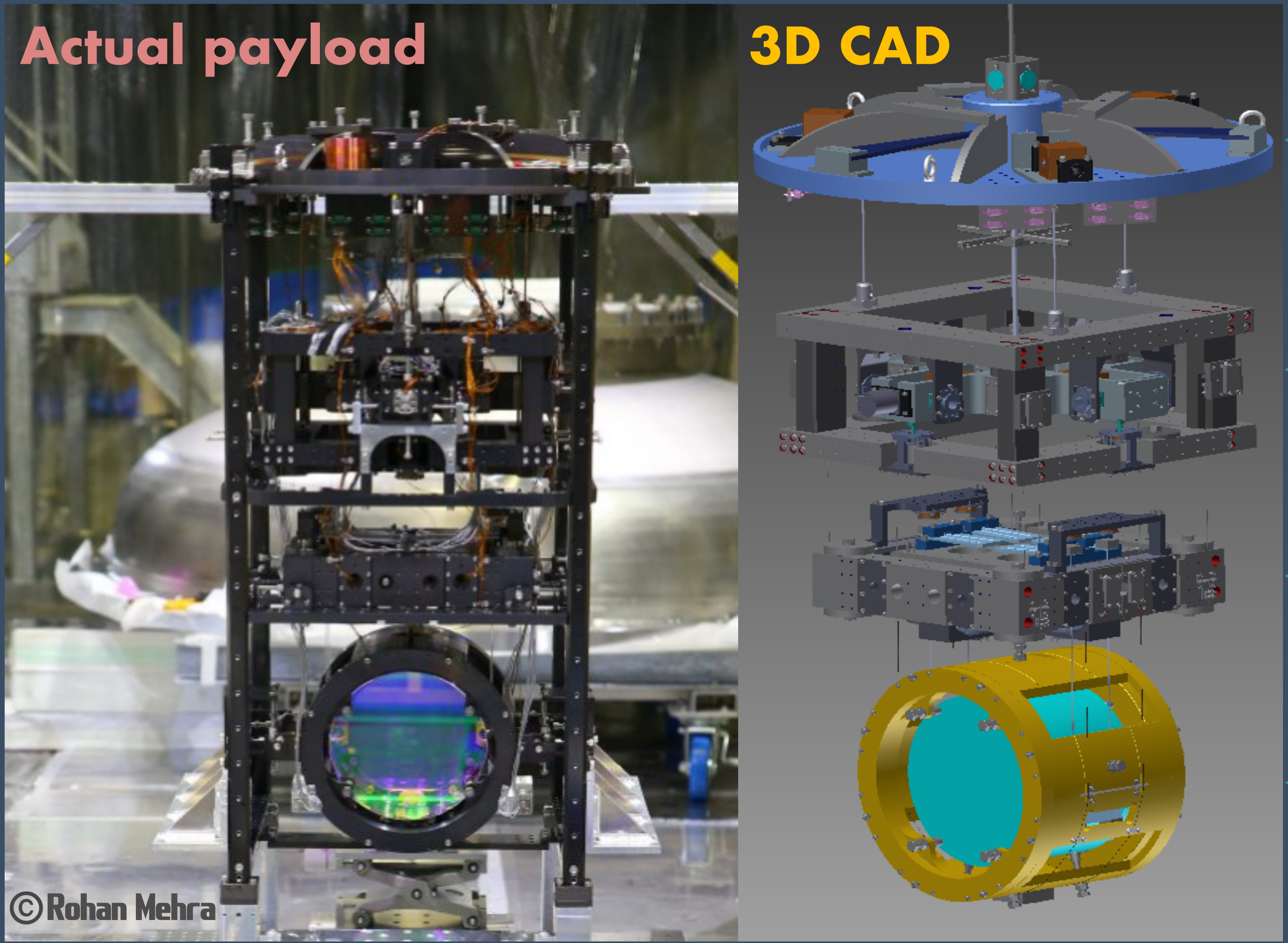


Cryostat & Cryogenic Payload



 [T. Ushiba, *et. al.*, Class. Quantum, 38, 085013, (2021)]

Cryogenic Payload



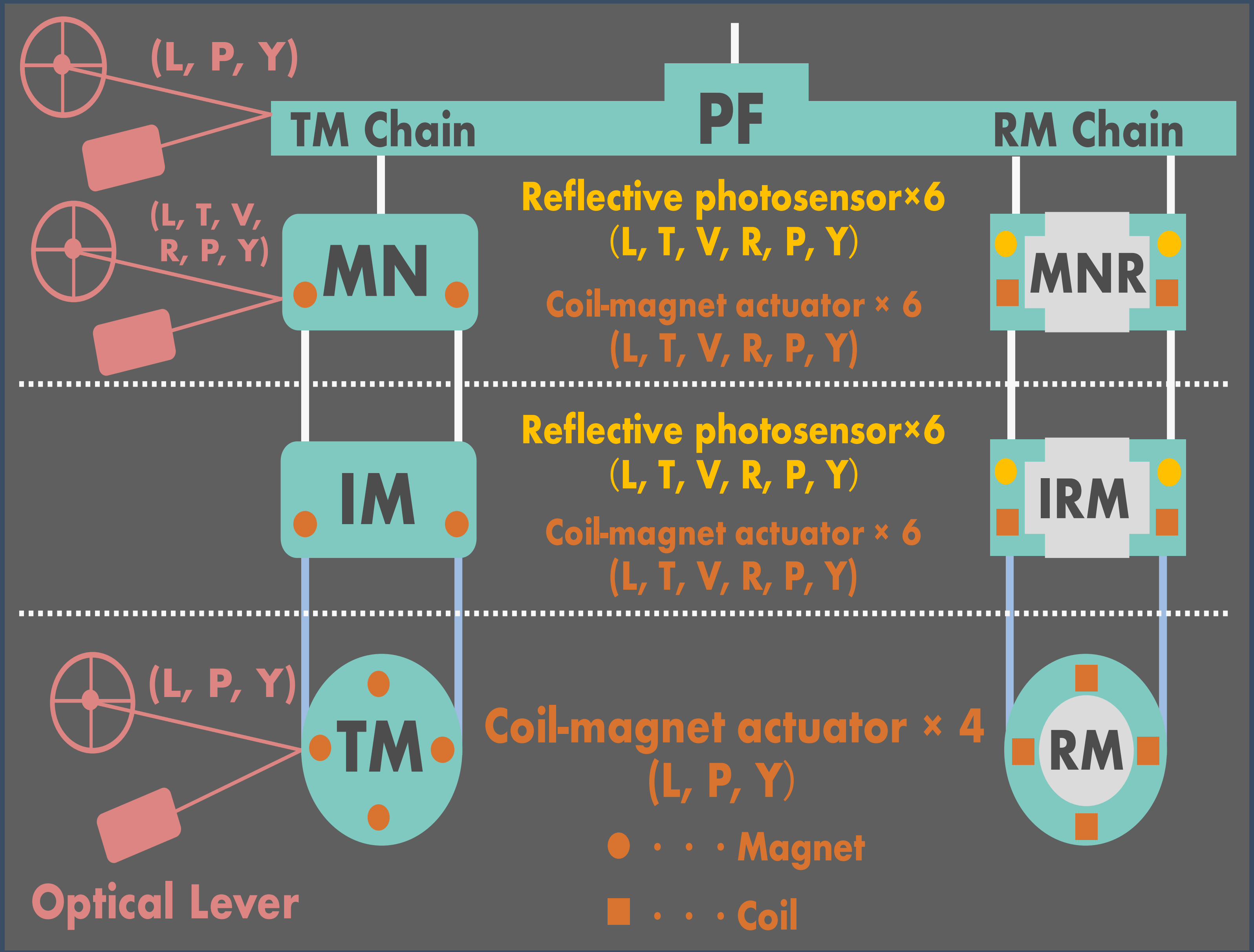
Platform (PF)

Marionette (MN) & Recoil mass

Intermediate mass (IM) & Recoil mass

Test mass (TM) & Recoil mass

Sensors & Actuators



Reflective photosensor

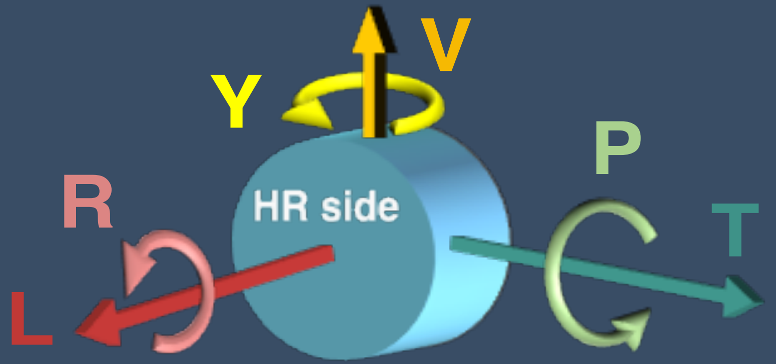
- Measure the relative displacement of the mass and recoil mass

Optical Lever

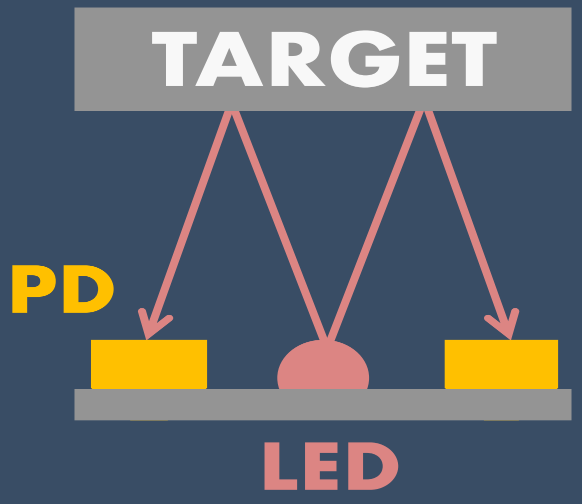
- Measure the movement of the mass relative to the ground

Coil-magnet actuator

- Apply the force to the mass



Reflective Photosensor

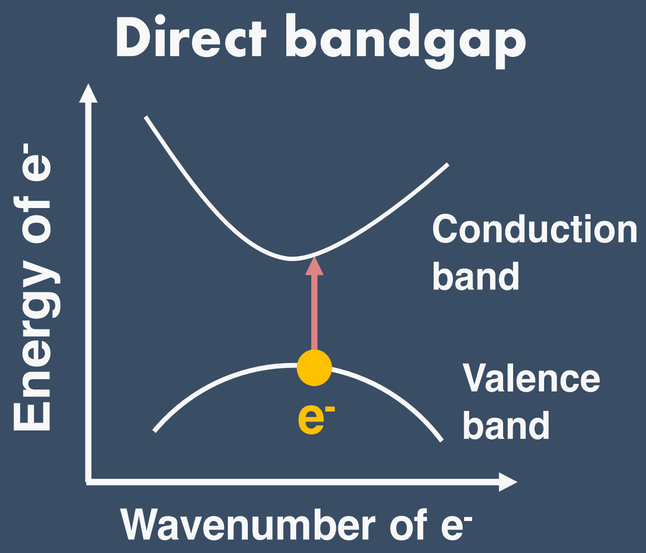


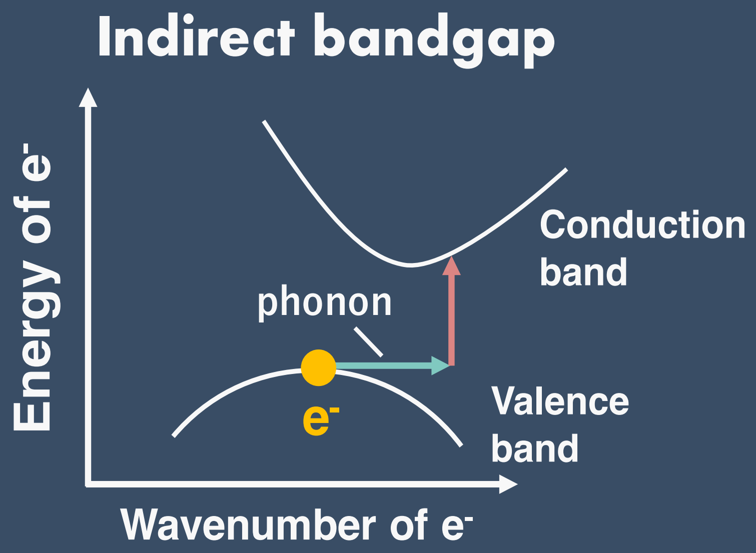
- Monitor the changes in relative distance as changes in light intensity
- Wide dynamic range for use at cryogenic temperature

PD: InGaAs LED: InGaAsP

⪢ Direct bandgap for cryogenic use

No energy assistance by phonons





AGENDA

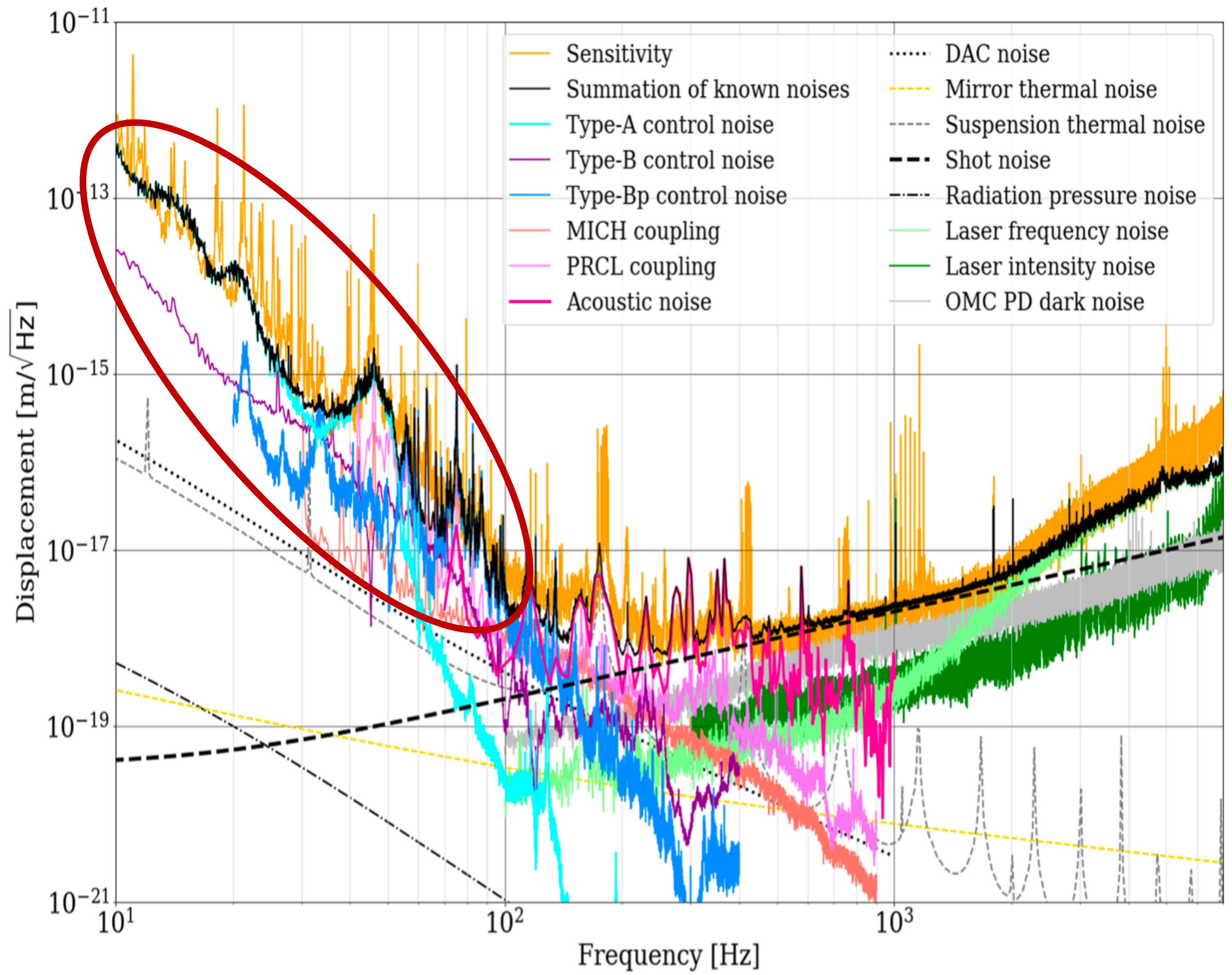
1 **KAGRA Suspensions**

damping control noise during O3GK & O4a

2 **Problems**

3 **Solutions (Future Works)**

KAGRA Sensitivity in O3GK



Observing run with GEO600

(Apr. 7 - Apr. 21, 2020)

Below 100 Hz

» Limited by **local damping control noise**

10 ~ 50 Hz: Type-A

50 ~ 100 Hz: Type-Bp

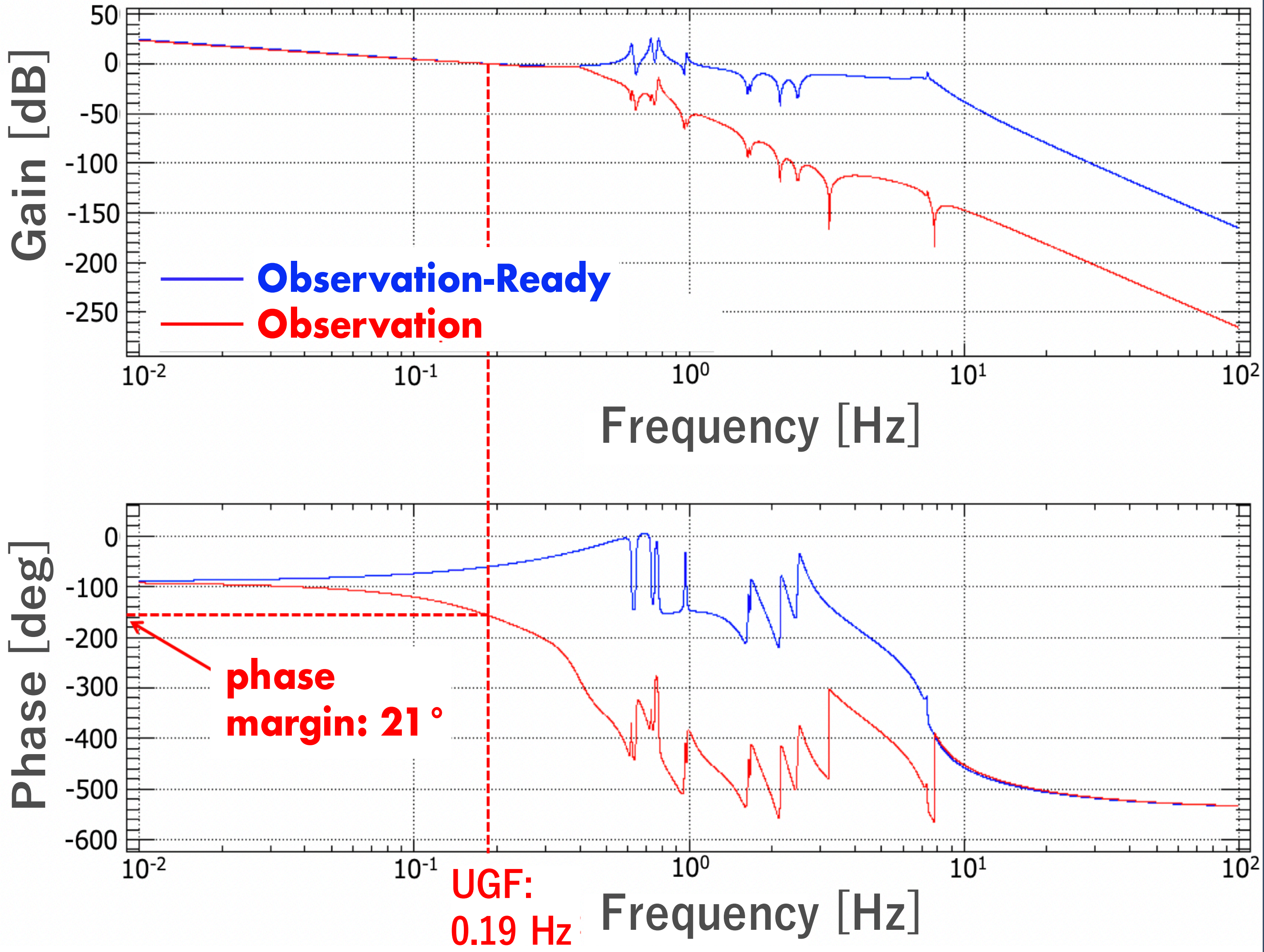
[H. Abe *et al*., PTEP, (2022)]

Improvement for O4a

For example...

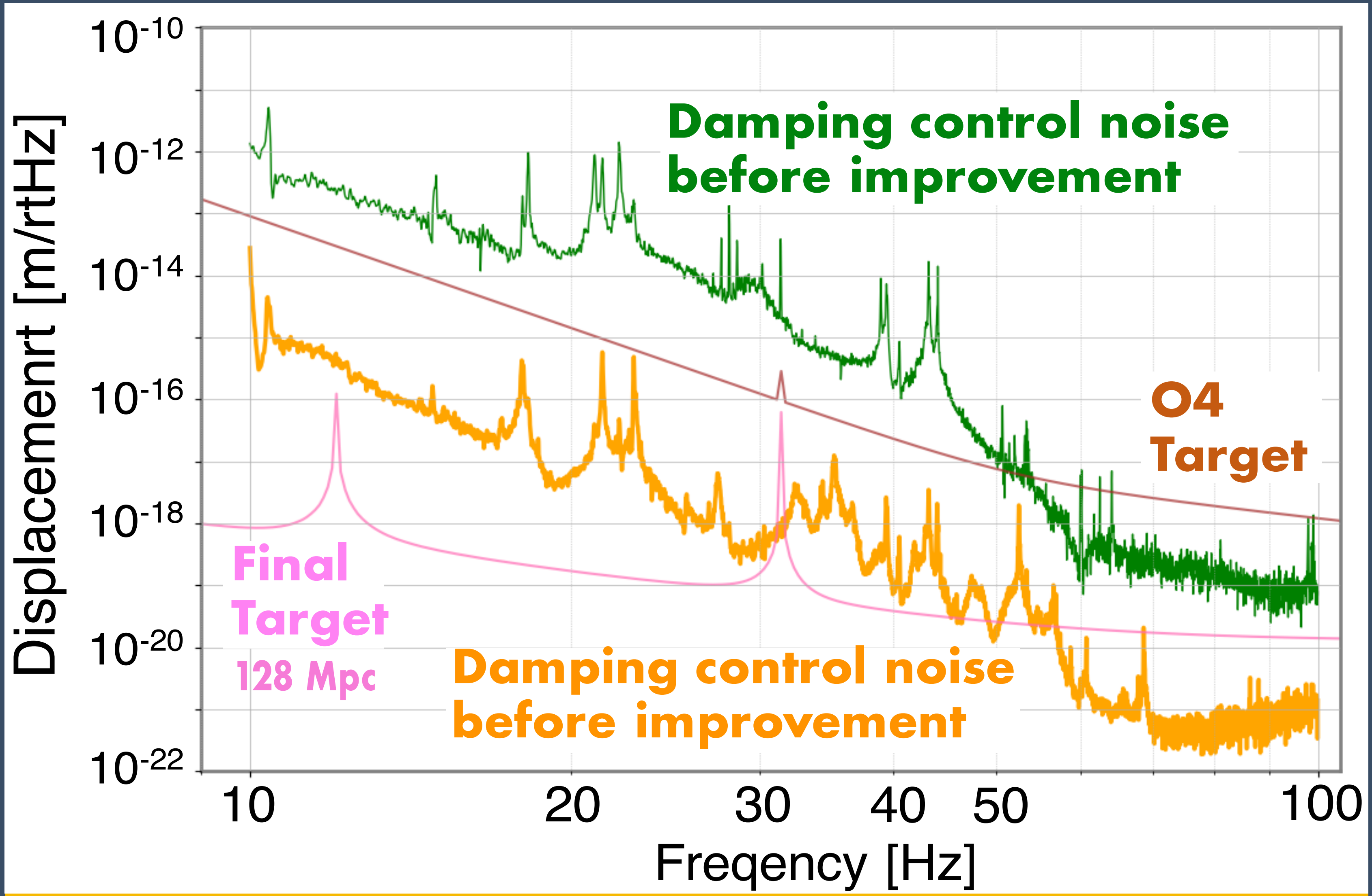
Before we lock the interferometer, damping control is on (Blue line)

After we lock the interferometer, we apply elliptic filter to reduce the gain above ~ 0.2 Hz for noise reduction (Red line)



ex) open loop TF of damping control ITMX P

Improvement for O4a

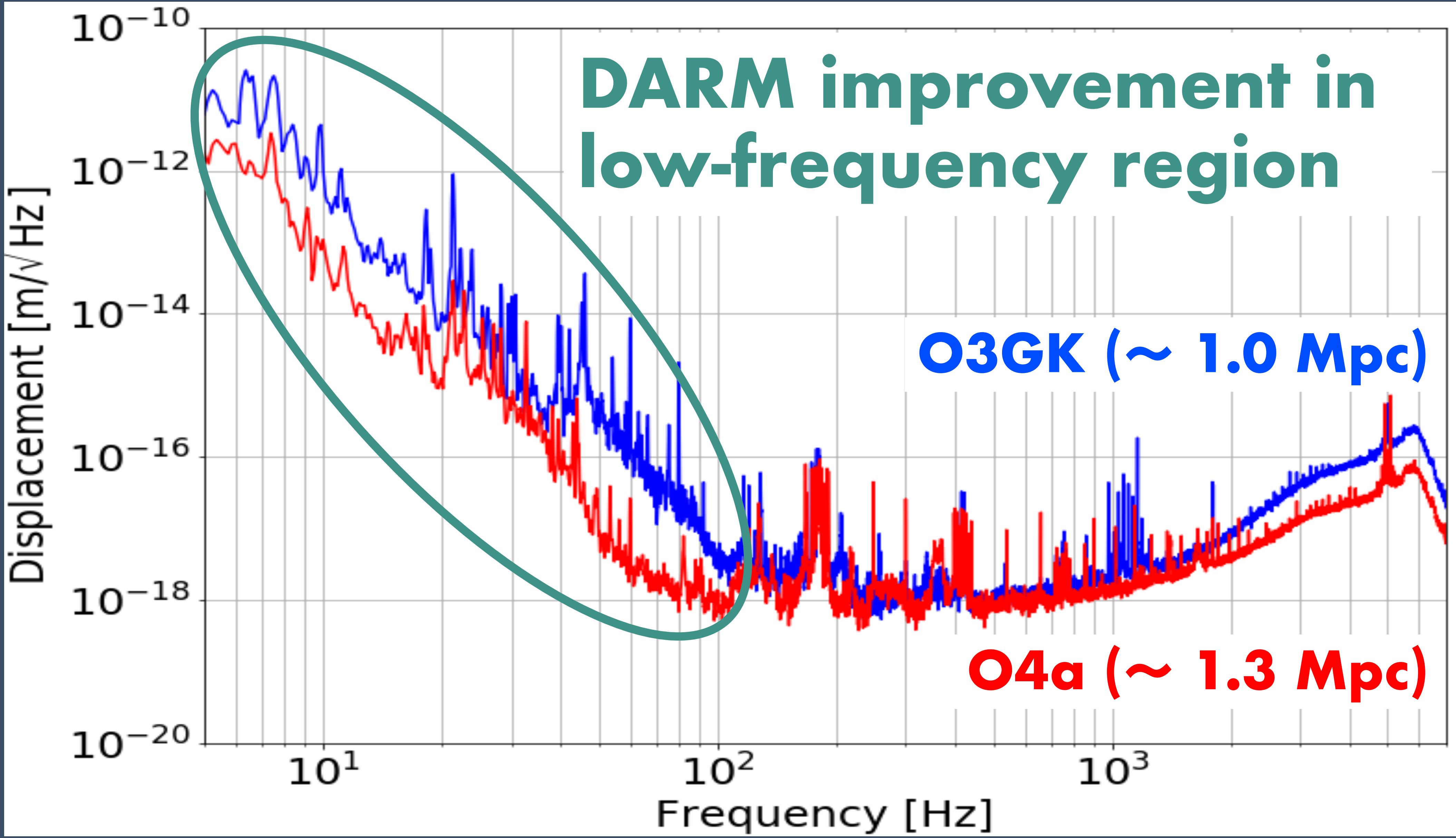


damping control noise for Type-A suspension (except for EX)

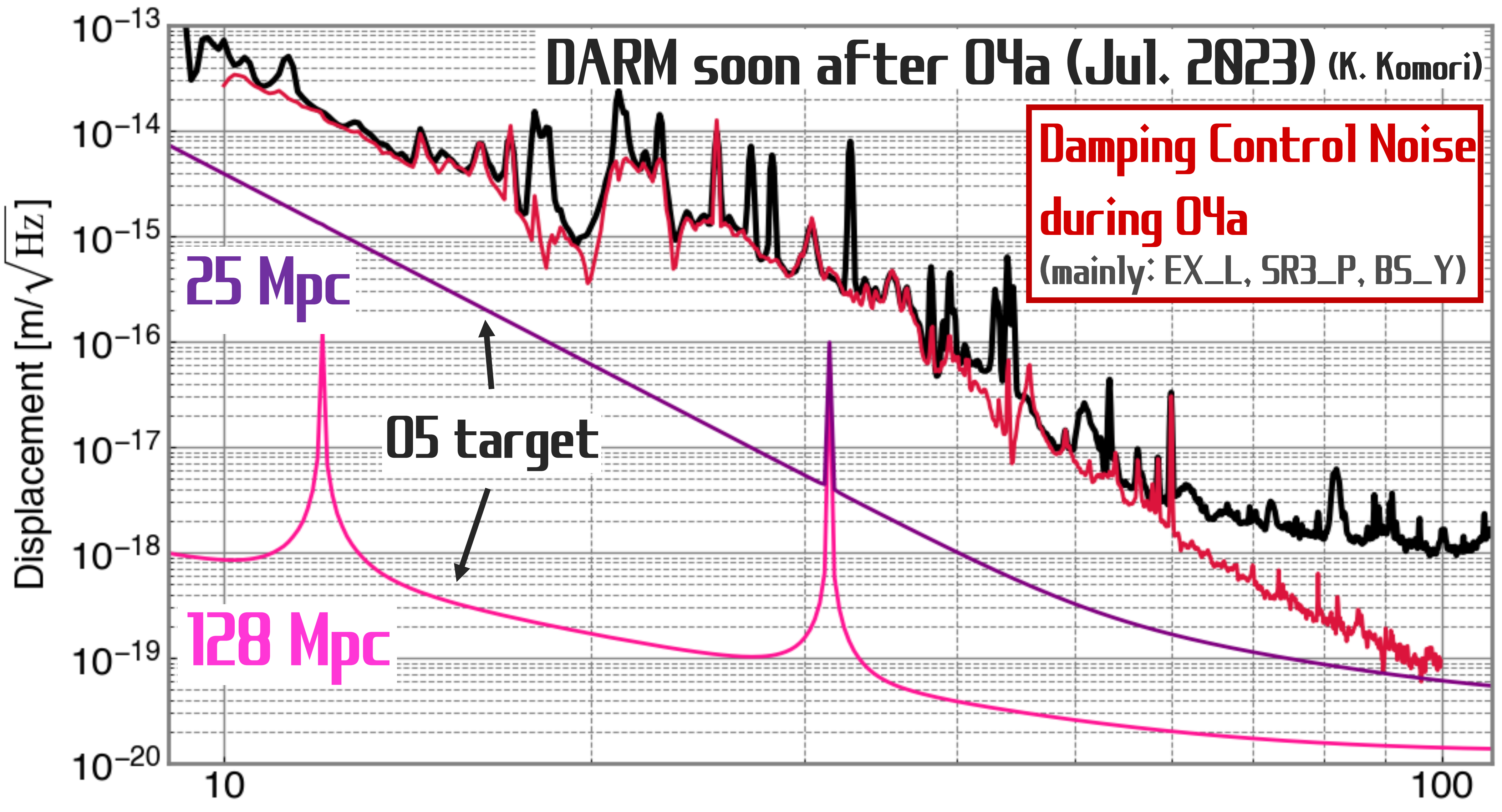
Some updates on control loop **(apply elliptical low-pass filter above 10 Hz in observation phase)** was done for Type-A suspension (except for ETMX)

As for the suspension of which we improved the loop, **damping control noise successfully reduced**

Improvement for O4a

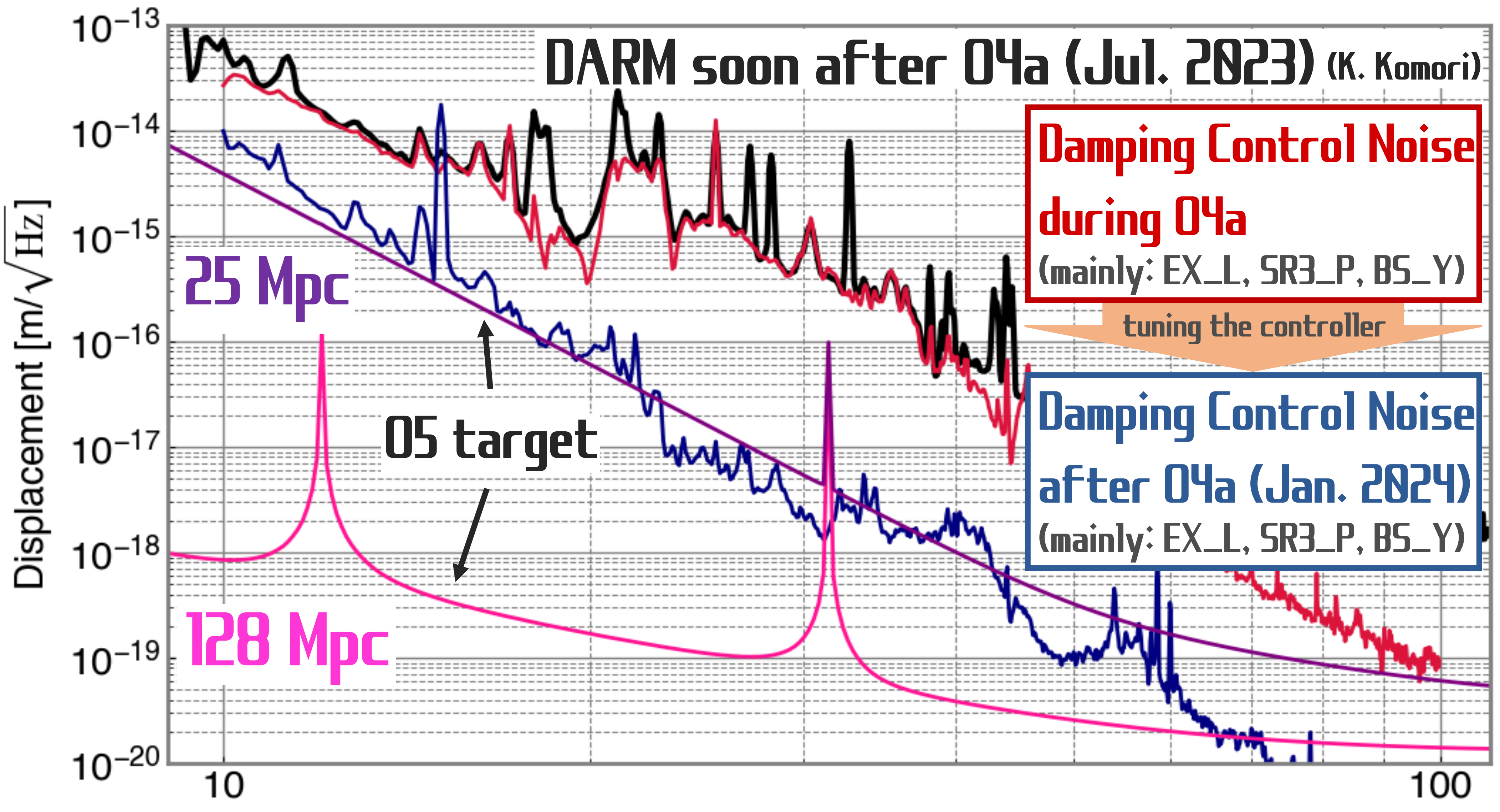


Problem - Damping Control Noise -



data is from JGW-T2113624-v3 (Y. Michimura)

Problem - Damping Control Noise -



data is from JGW-T2113624-v3 (Y. Michimura)

AGENDA

1 KAGRA Suspensions

2 Problems

3 Solutions (Future Works)

How can we reduce damping control noise?

"Better" sensor

- Less noise sensor (compare to reflective photosenser)

under consideration to develop

*we should consider cryogenic operation

"Better" control

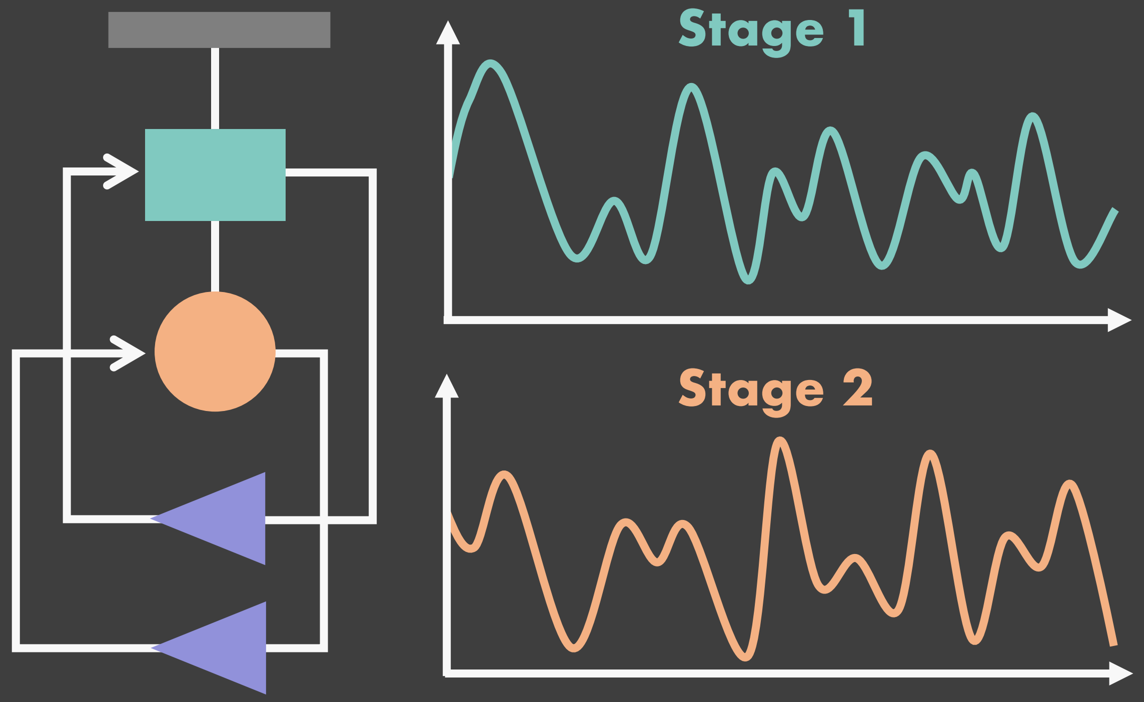
- Optimize the trade-off between damping performance & control noise

- tuning the controller
- some advanced techniques

etc…

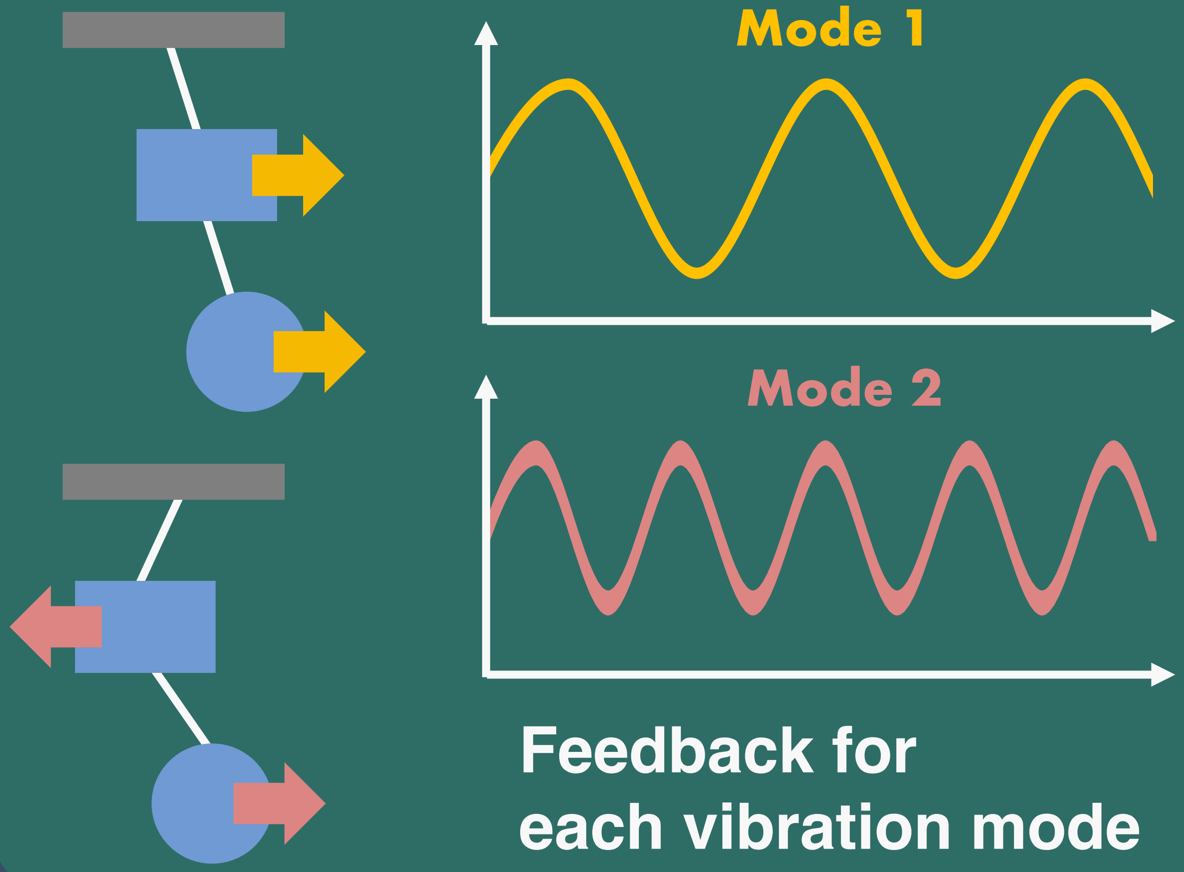
Modal Damping
- one of the candidates for improvement -

Conventional control in KAGRA



Feedback for each stage

Modal damping control



Feedback for each vibration mode

Modal Damping
- one of the candidates for improvement -

Optimization of
damping control
for multi-DoF system
is complex

»

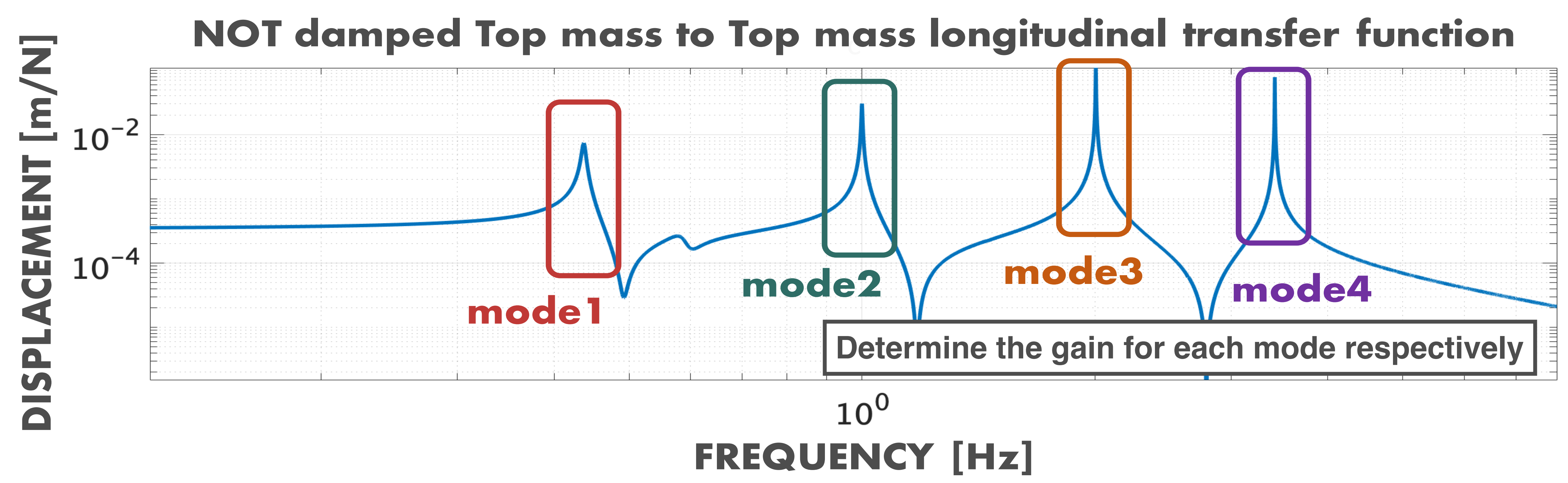
Single DoF resonance problem

▼

Controller & its optimization should be simpilified

Decouple the oscillations of the system into modes

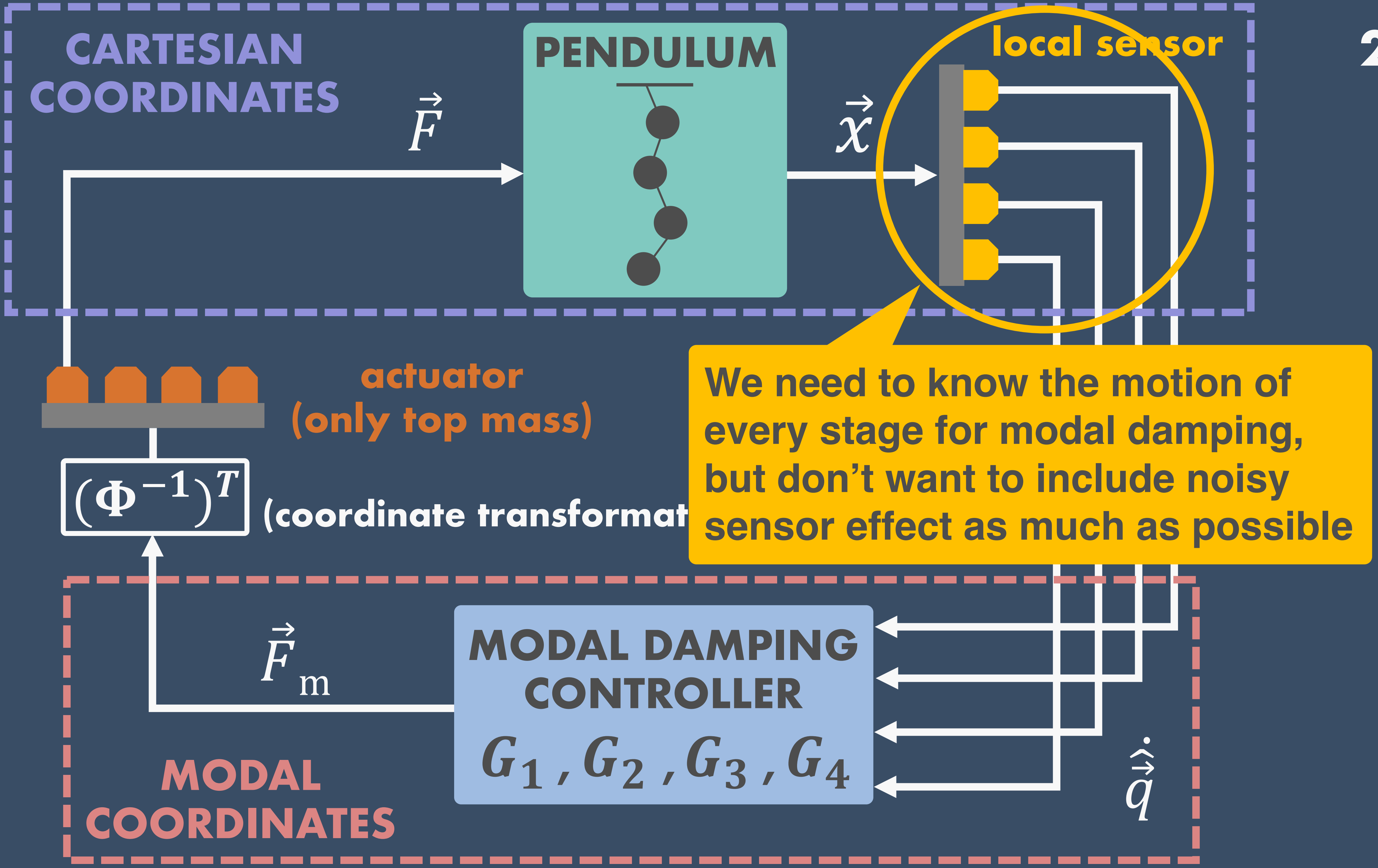
Gain tuning becomes easy

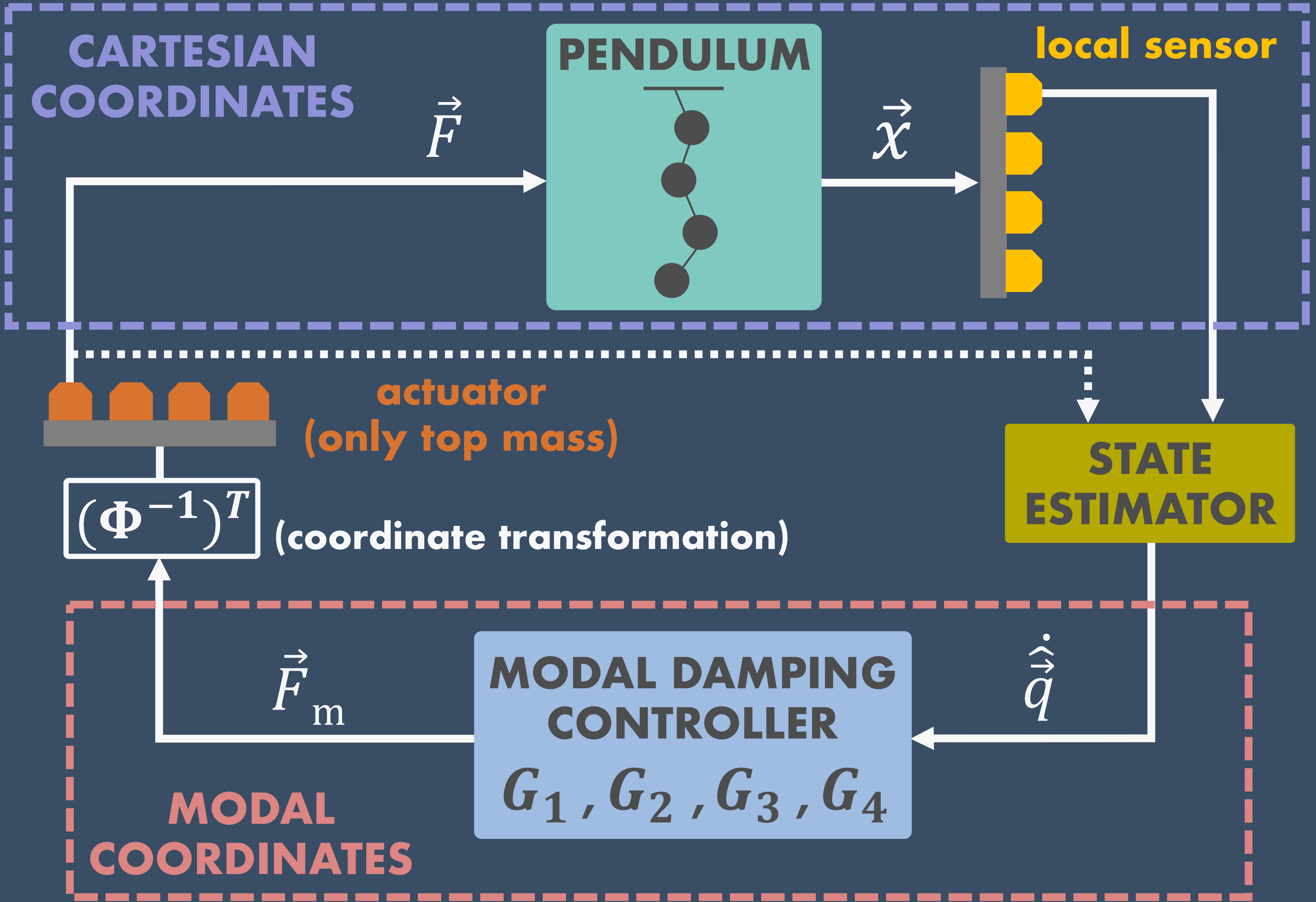


(For example...)

Gain can be increased for modes that require damping and decreased for those that don't require **for noise performance**

→ Automation according to the daily seismic noise is also possible


CARTESIAN
COORDINATES
$\vec{F}$
PENDULUM
$\vec{x}$
local sensor
actuator
(only top mass)
$(\Phi^{-1})^T$
(coordinate transformat
We need to know the motion of every stage for modal damping, but don't want to include noisy sensor effect as much as possible
$\vec{F}_{\mathrm{m}}$
MODAL DAMPING
CONTROLLER
G_1, G_2, G_3, G_4
$\dot{\hat{\vec{q}}}$
MODAL
COORDINATES


CARTESIAN COORDINATES
PENDULUM
local sensor
$\vec{F}$
$\vec{x}$
actuator
(only top mass)
STATE ESTIMATOR
$(\Phi^{-1})^T$
(coordinate transformation)
$\vec{F}_{\mathrm{m}}$
MODAL DAMPING CONTROLLER
G_1, G_2, G_3, G_4
$\hat{\dot{\vec{q}}}$
MODAL COORDINATES

State Estimator

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$$

$$\hat{y} = C\hat{x} \qquad \hookrightarrow u = -K\hat{x}$$

System

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

Estimator error: $\dot{e} = \dot{x} - \dot{\hat{x}} = (A - LC)e$

State space of closed loop system

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}$$

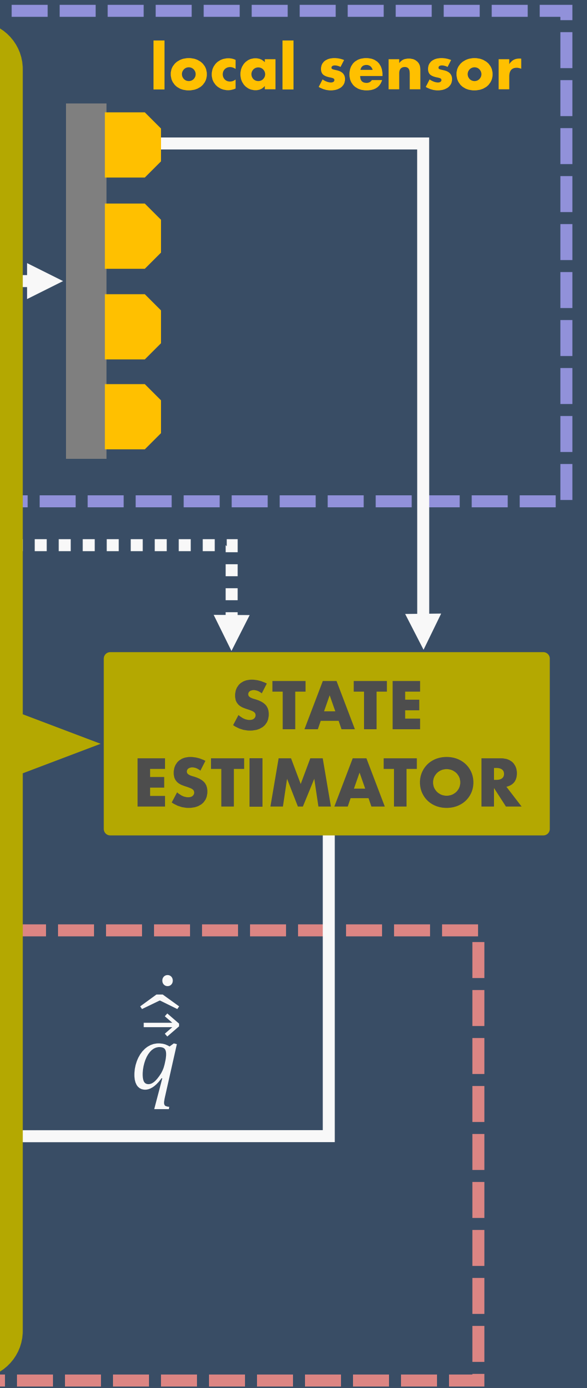
Designed with LQR

$$J = \int_0^\infty (x^\mathrm{T} Q x + u^\mathrm{T} R u)\mathrm{d}t$$

Weight

Q : for estimation error

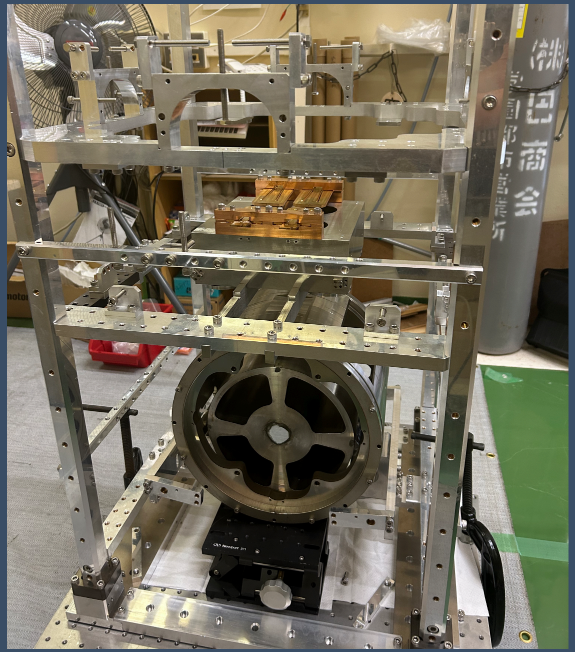
R : for sensor output



Simulation

modal damping control with state estimator for dummy cryogenic payload

Dummy cryogenic payload →
(in preparation)

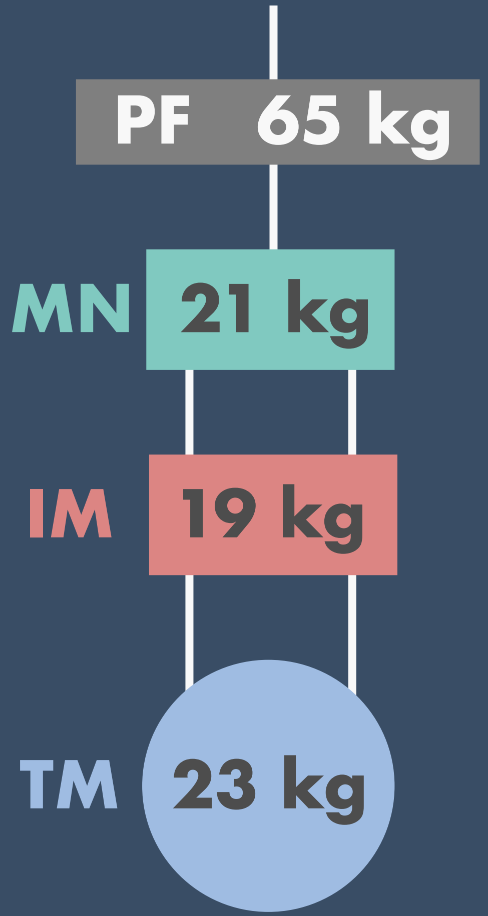


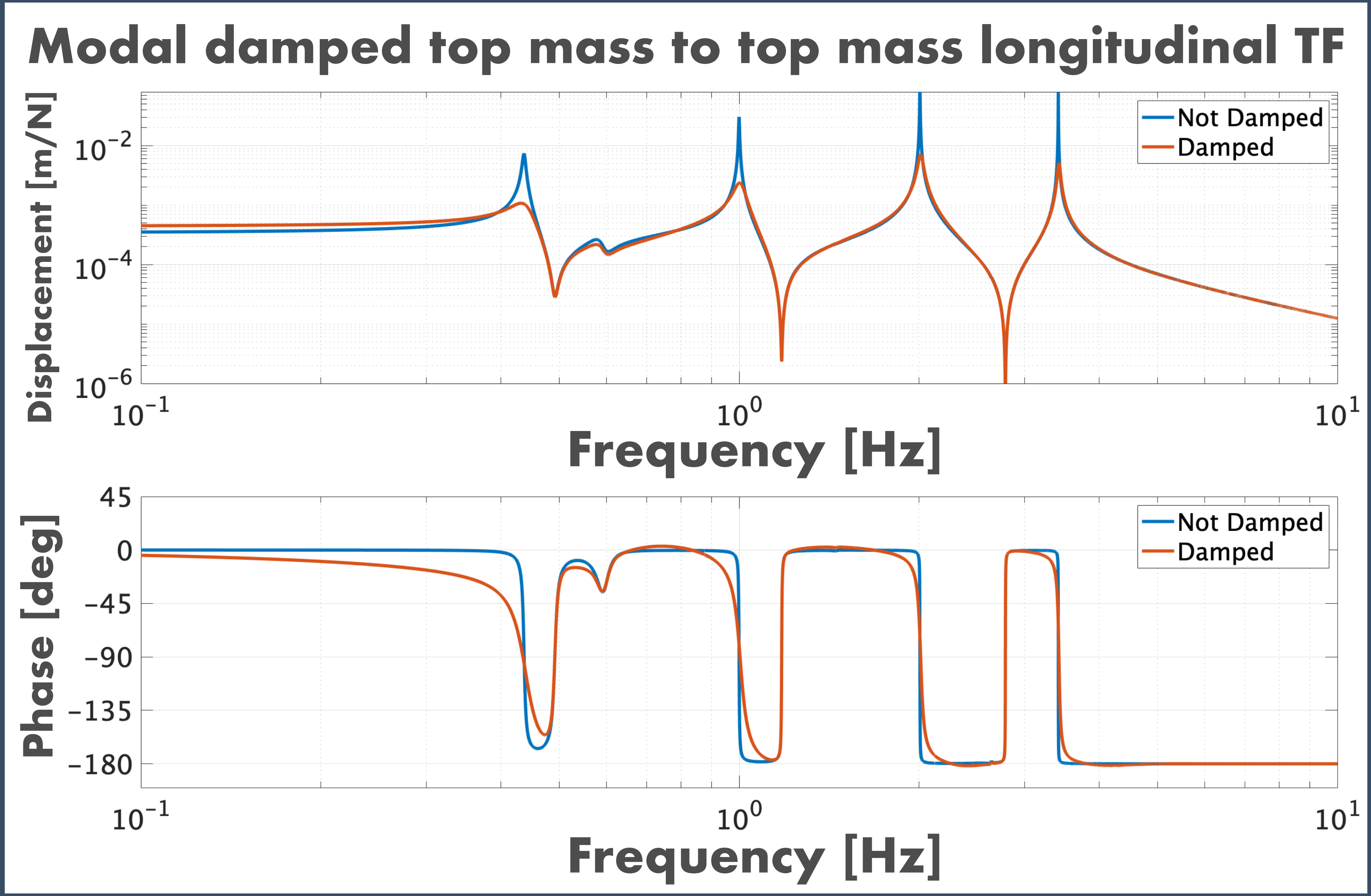
[*]

was used
to construct the model of cryogenic payload
and get its state space matrix

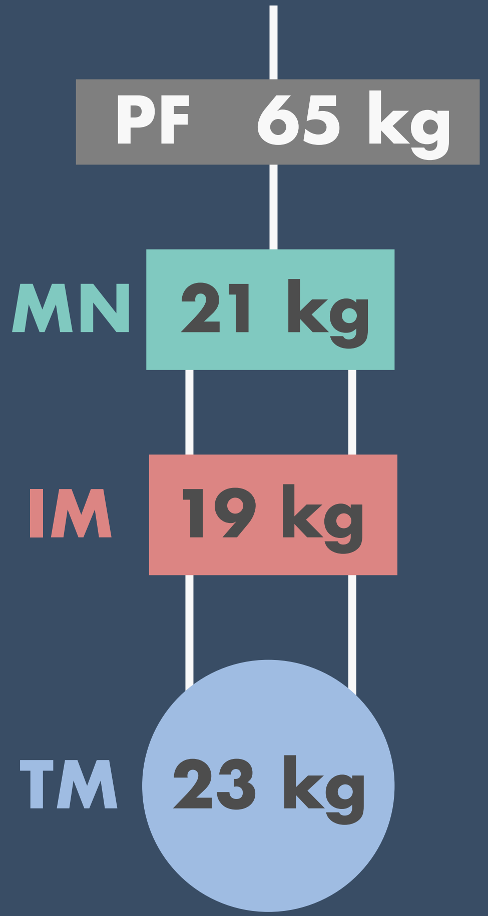
[* T. Sekiguchi, JGW-T1503729, (2015)]

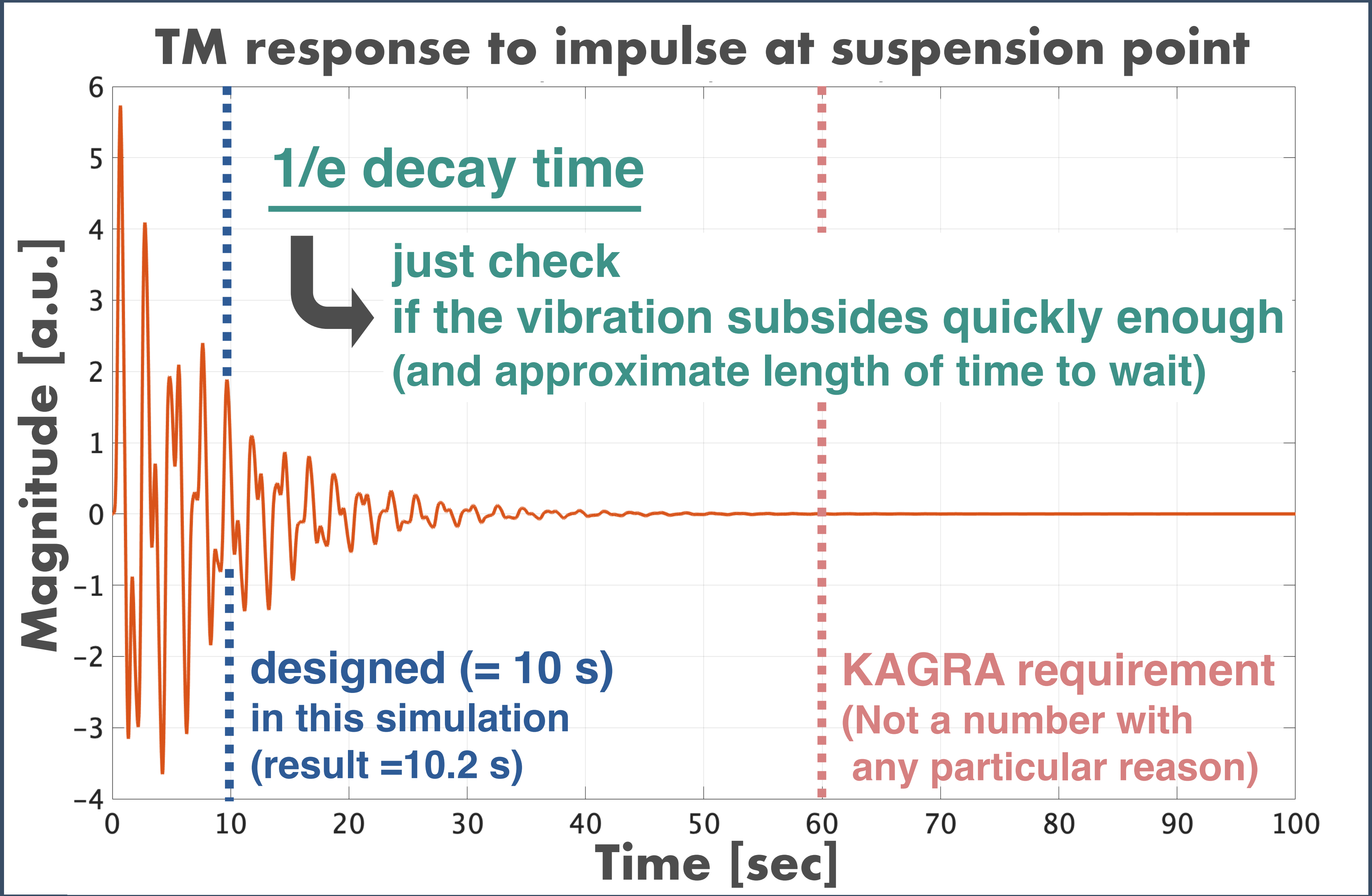
Result of Simulation



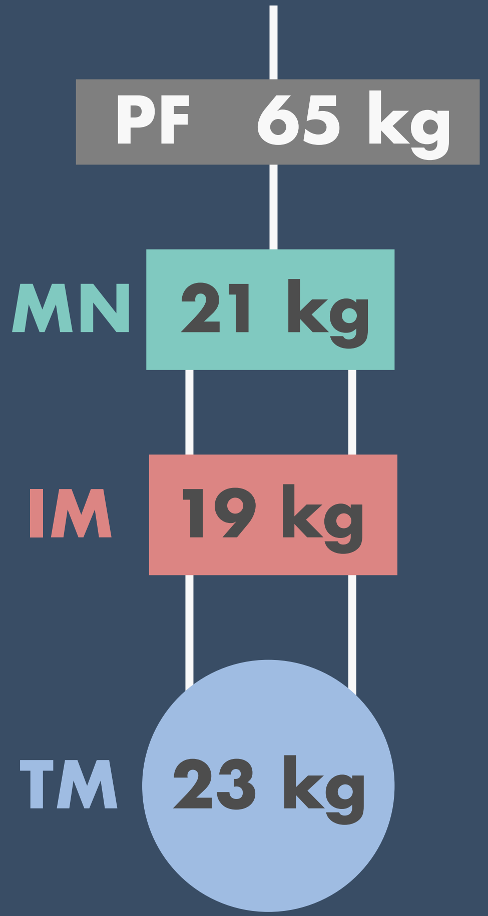


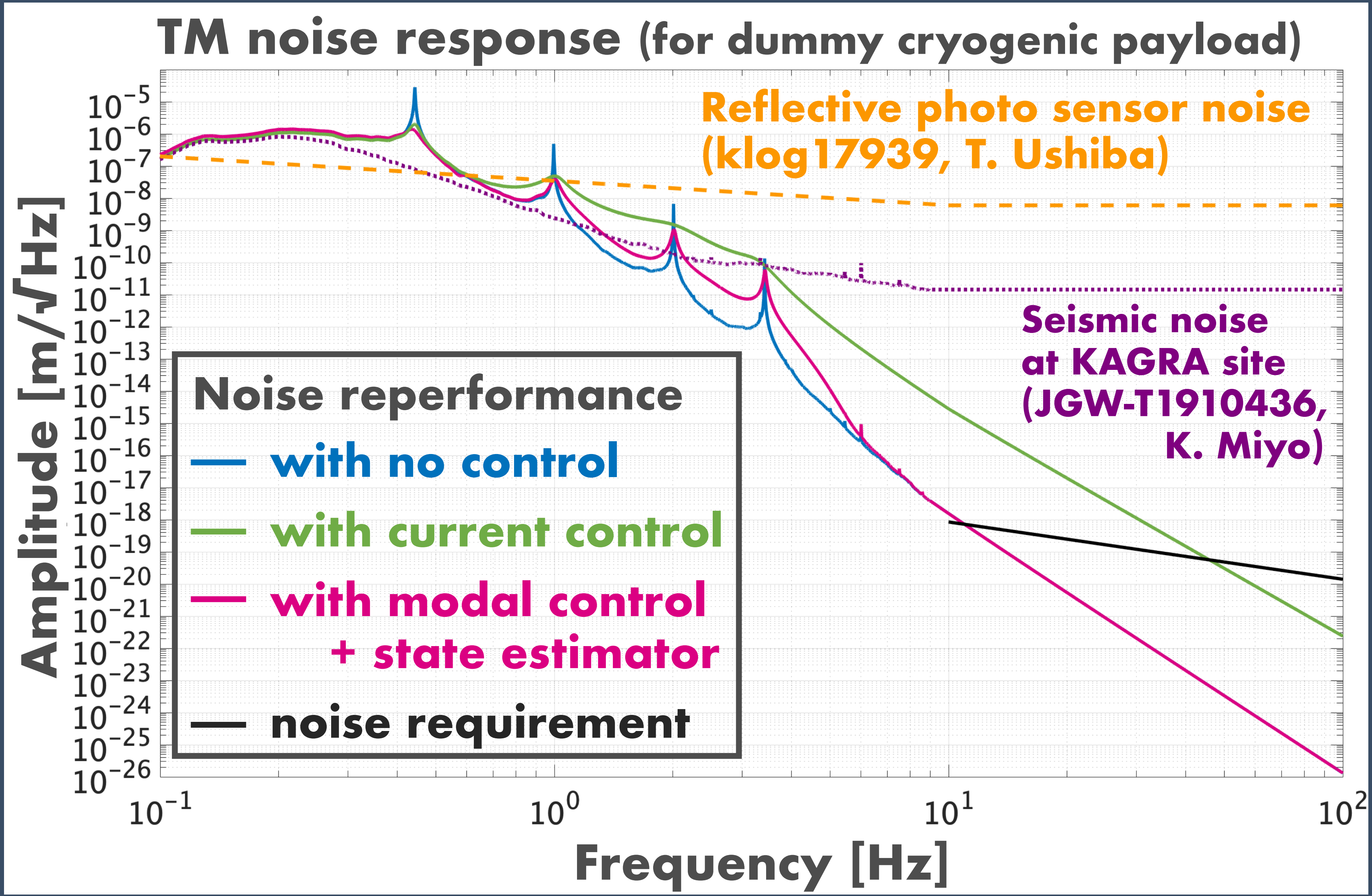
Result of Simulation





Result of Simulation





Conclusion & Outlook

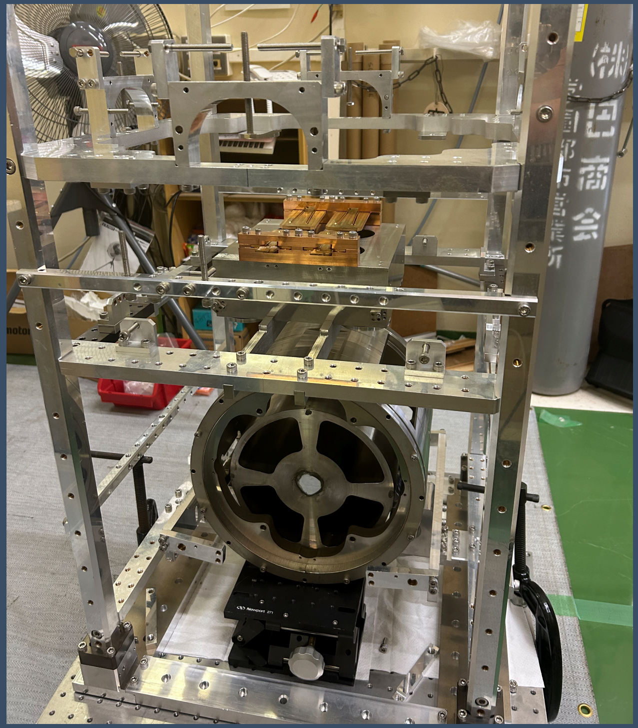
- ✓ **Suspension damping control** has been updated from O3GK.

- ✓ However, **it still need to be improved to achieve O5 target sensitivity**

- ✓ Simulation of new control and preparation of test are ongoing

  Dummy cryogenic payload (in preparation) →

  We are considering to develop new cryogenic sensor and test together




THANK
YOU!

BACKUP SLIDES

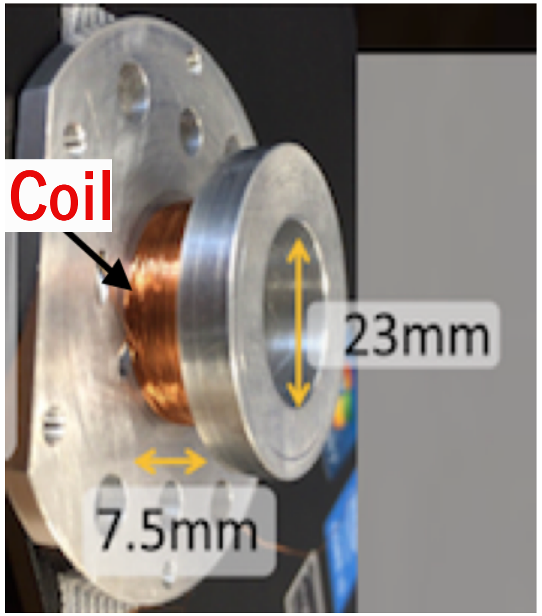
Coil-magnet Actuator

Coil (RM chain) + SmCo magnet (TM chain)

Coils are also vibration isolated

A magnetic field is generated by passing an electric current through the coil

→ Apply the force to the magnet

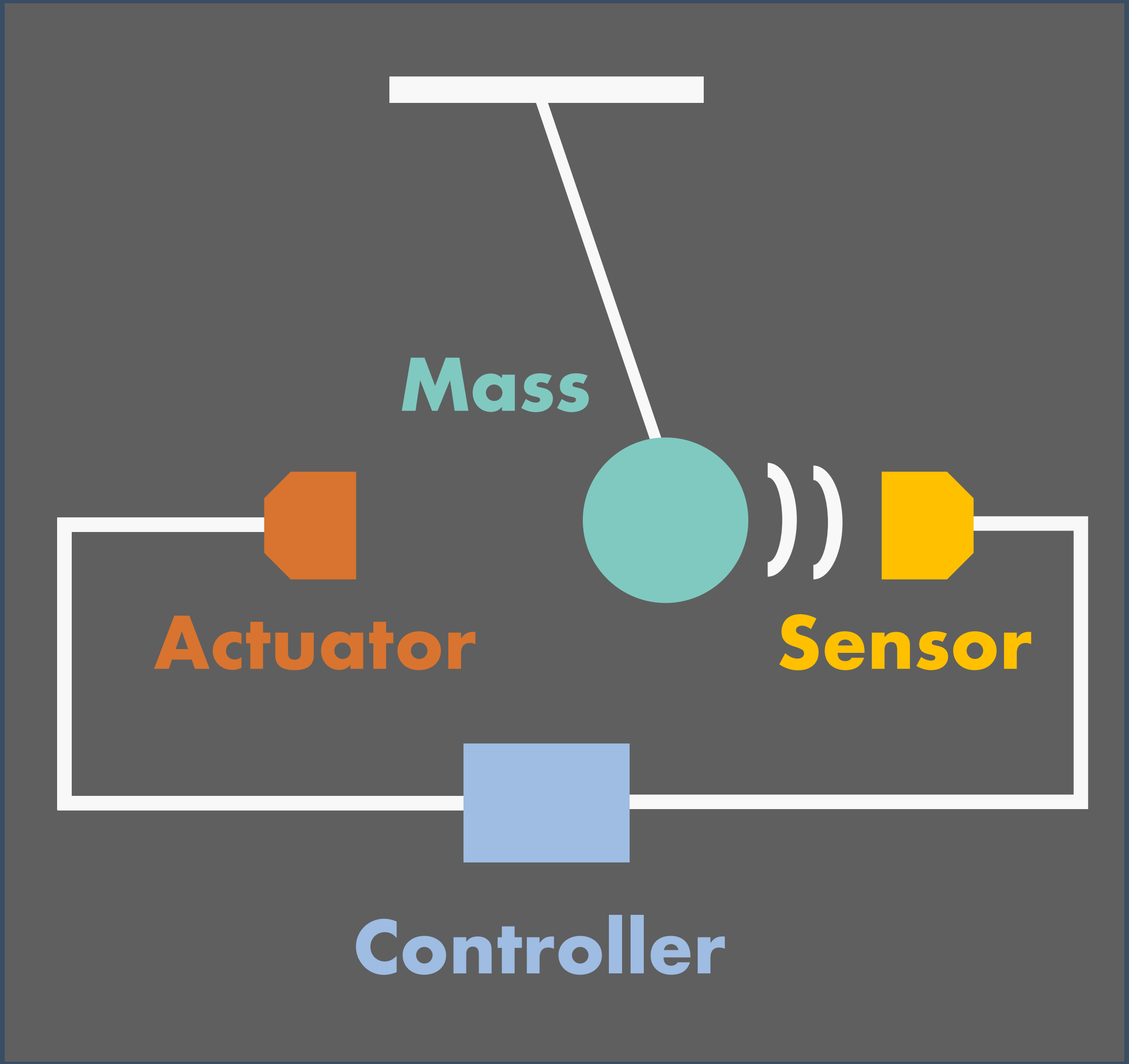


Magnetization does not decrease significantly even at cryogenic temperatures

Magnet

standoff

Local Damping Control



01 Detect the motion of the mass by local sensor

02 Send signal to the actuator through the controller

03 Applies a force proportional to the velocity of the mass to suppress the vibration

Calculation for Mode Decomposition

- Cartesian EoM of suspension

$$\mathbf{M}\ddot{\vec{x}} + \mathbf{K}\vec{x} = \vec{F}$$

- Coordinate transformation

$$\vec{x} = \mathbf{\Phi}\vec{q}$$

- **Modal Equation**

$$\mathbf{M}_{\mathrm{m}}\ddot{\vec{q}} + \mathbf{K}_{\mathrm{m}}\vec{q} = \vec{F}_{\mathrm{m}}$$

*subscript "m" means they are in modal basis

$\vec{x}$:displacement $\vec{q}$:modal displacement $\vec{F}$:Control force

$\mathbf{M}$:Mass matrix $\mathbf{K}$:Stiffness matrix

$\mathbf{\Phi}$:Eigen vector of matrix $\mathbf{M}^{-1}\mathbf{K}$

For the aplication to real system, transformation related to sensor/actuator signals are also required

How to Decompose the Mode

Equation of Motion: $\mathbf{M}\ddot{x} + \mathbf{K}x = 0$

$\Downarrow$ Fourier transform

$$(-\omega^2\mathbf{M} + \mathbf{K})\tilde{x}(\omega) = 0$$

$\Downarrow$ $\lambda = \omega^2, \phi = \tilde{x}$

$$(\mathbf{K} - \lambda\mathbf{M})\phi = 0$$

$\Downarrow$

Eigenvector: ϕ , Eigenvalue: λ

How to Decompose the Mode

$\mathbf{M}$: positive definite matrix

$\mathbf{K}$: positive semi-definite matrix

real symmetric

- eigenvectors corresponding to distinct eigenvalues are always orthogonal to one another

- eigenvectors of ϕ are new basis **(modal basis)**

State Estimator

State space equation

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

same dynamics

$$u = -K\hat{x}$$

x: system's state (position, velocity, etc...)

u: input

y: measurement

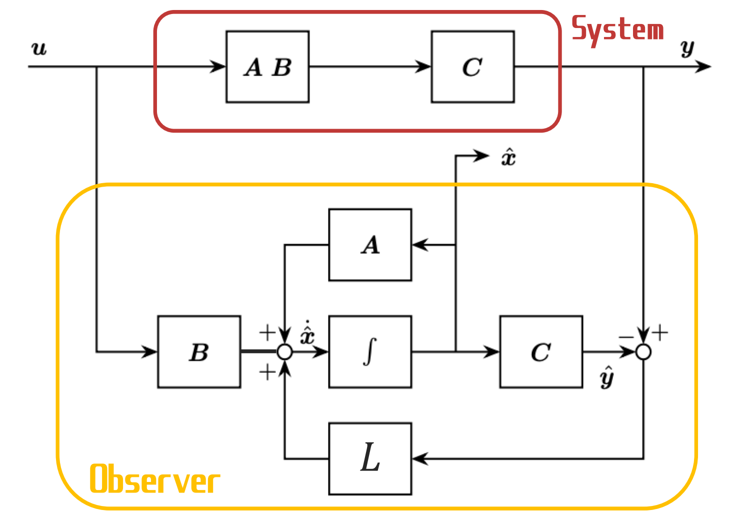
State estimator

$$\dot{\hat{x}} = A\hat{x} + Bu$$
$$\hat{y} = C\hat{x}$$

estimated state will be included in the feedback control

Given the same input signal u, and produce the estimation $\hat{x}$

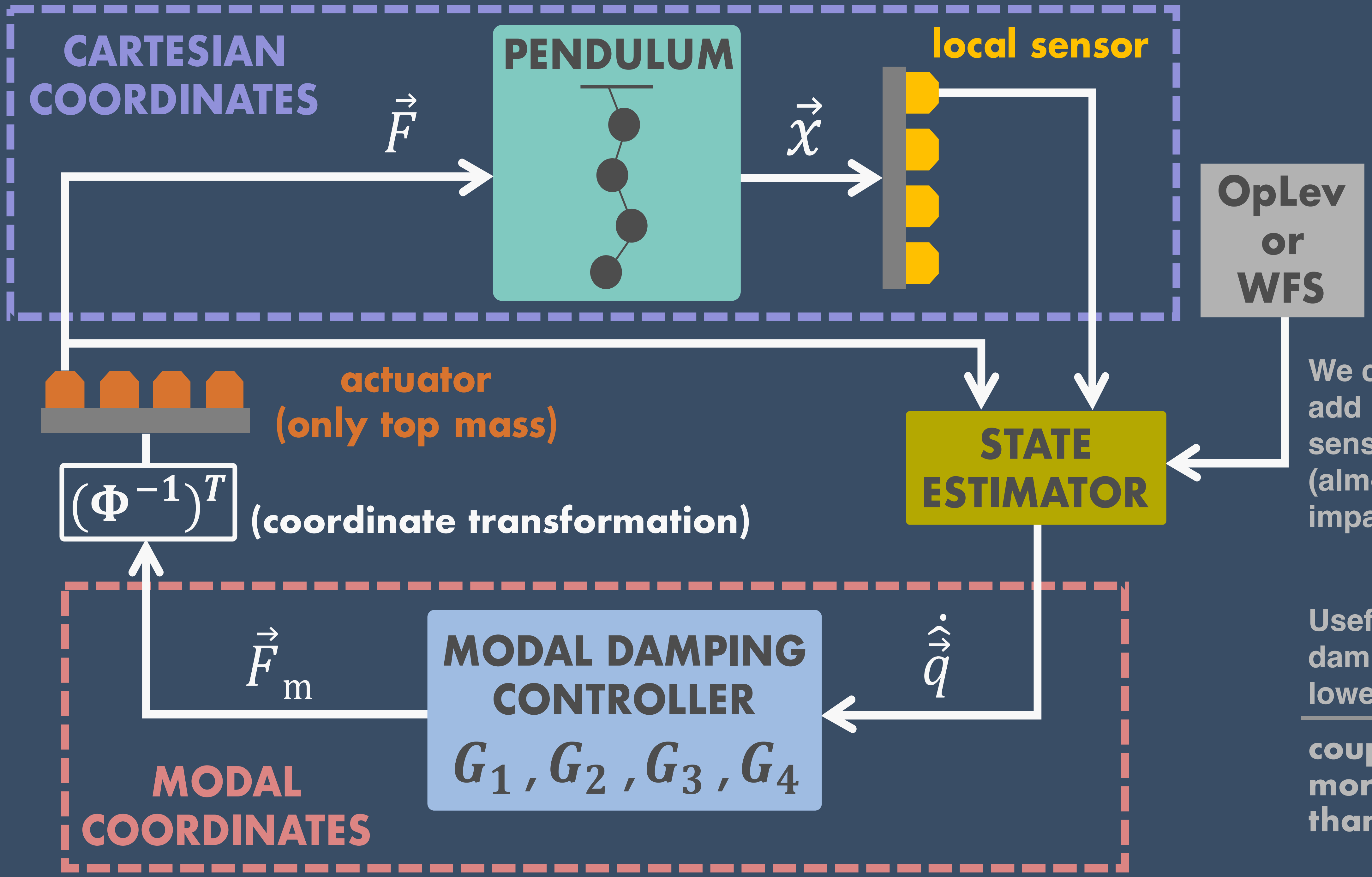
The estimate of output $\hat{y}$ is compared to y



Error: $e = y - \hat{y} = y - C\hat{x}$

To minimize e, a term proportional to the error is added to $\dot{\hat{x}}$

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$$: State Estimator



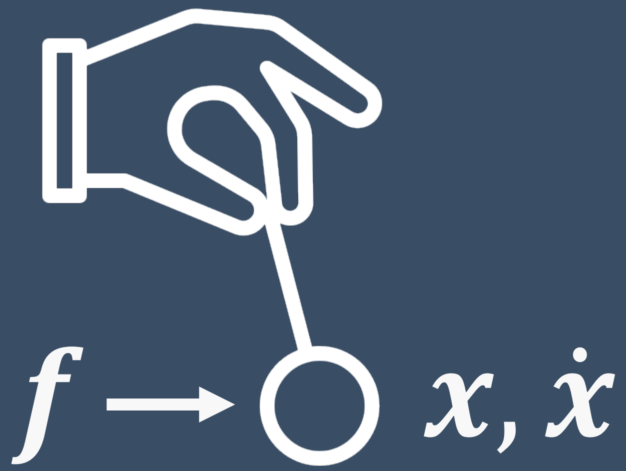
We can easily add in other sensors with (almost) no impact on stability

Useful for damping of the lowest freq. mode

couple more to test mass than to top mass

What is Linear Quadratic Regulator?

Automatic algorithm which finds **optimal** feedback for linear systems in state-space form



x : Displacement State

$\dot{x}$: Velocity State

f : Actuation

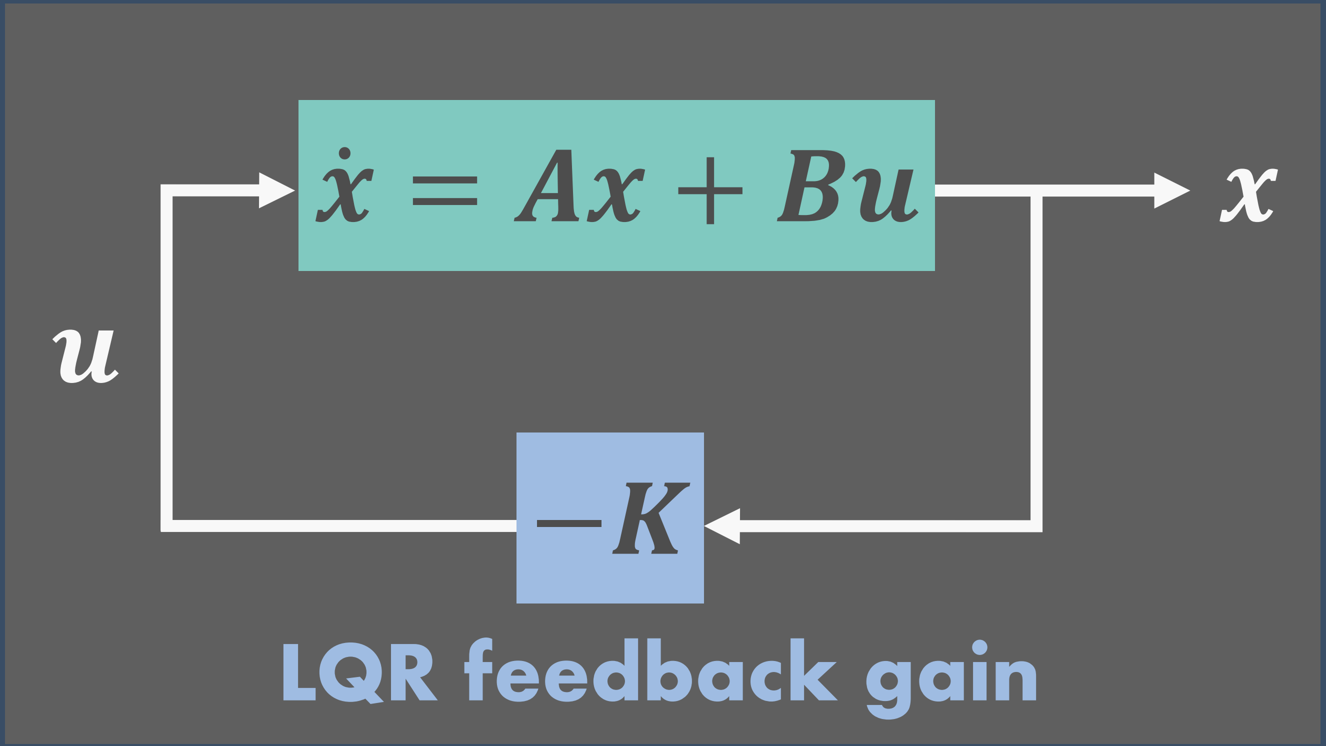
What is Linear Quadratic Regulator?

Automatic algorithm
which finds **optimal** feedback
for linear systems in state-space form

Minimize the size of state vector

- **in the least time**
- **with minimum control effort**

System



n states, m inputs

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$$

$$A: n \times n, B: n \times m,$$
$$K: m \times n$$

Assumption

01 (Completely) Controllable

02 We have knowledge of all states x

Our Goal Here

- make the size of x very small

$$x^{\mathrm{T}}x \to 0$$

- make the size of u very small

(to minimize the noise / keep actuator in range)

$$u^{\mathrm{T}}u \to 0$$

Total Cost

$$J = \int_0^{\infty} (x^{\mathrm{T}}x + u^{\mathrm{T}}u)\mathrm{d}t$$

Cost Function

$$J = \int_0^{\infty} (x^{\mathrm{T}} x + u^{\mathrm{T}} u) \mathrm{dt}$$

We want $x^{\mathrm{T}} x \to 0$ & $u^{\mathrm{T}} u \to 0$ are true at all time

care about the size of some states / actuators than others

$$J = \int_0^{\infty} (x^{\mathrm{T}} Q x + u^{\mathrm{T}} R u) \mathrm{dt}$$

$$Q = Q^{\mathrm{T}} \geq 0, R = R^{\mathrm{T}} \geq 0$$

Q, R: weighting function

Minimize this with

- Linear algebra
- Calculs of variations

How can we find the feedback gain with LQR?

Solve ARE below

(Algebraic Riccati Equation)

$$PA + A^{\mathrm{T}}P - PBR^{-1}B^{\mathrm{T}}P + Q = 0$$

- Calculate P
- Feedback gain $K = R^{-1}B^{\mathrm{T}}P$

  K minimizes J
- Control input $u = -R^{-1}B^{\mathrm{T}}Px$

LQR in Frequency Domain

$$J = \int_0^{\infty} (x^{\mathrm{T}} \boldsymbol{Q} x + u^{\mathrm{T}} \boldsymbol{R} u)\mathrm{d}t$$

Parseval's theorem

$$J = \frac{1}{2}\int_{-\infty}^{\infty} (x(-j\omega)^{\mathrm{T}} \boldsymbol{Q} x(j\omega) + u(-j\omega)^{\mathrm{T}} \boldsymbol{R} u(j\omega))\mathrm{d}\omega$$

we want treat these in frequency region

LQR in Frequency Domain

$$J = \frac{1}{2}\int_{-\infty}^{\infty} \left(x(-j\omega)^{\mathrm{T}} \boldsymbol{Q} x(j\omega) + u(-j\omega)^{\mathrm{T}} \boldsymbol{R} u(j\omega)\right) \mathrm{d}\omega$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \left(\underline{\bar{x}(-j\omega)^{\mathrm{T}} \bar{x}(j\omega) + \bar{u}(-j\omega)^{\mathrm{T}}\, \bar{u}(j\omega)}\right) \mathrm{d}\omega$$

Filtering function

$$\dot{\chi} = E_x \chi + F_x x$$
$$\dot{\mu} = E_u \mu + F_u u$$

Filtered states

$$\bar{x} = G_x \chi + H_x x$$
$$\bar{u} = G_u \mu + H_u u$$

LQR in Frequency Domain

$$J = \frac{1}{2}\int_{-\infty}^{\infty} (\bar{x}(-j\omega)^{\mathrm{T}}\bar{x}(j\omega) + \bar{u}(-j\omega)^{\mathrm{T}}\bar{u}(j\omega))\mathrm{d}\omega$$

Parseval's theorem

$$J = \int_{0}^{\infty} (x^T H_x^T H_x x + 2\, x^T H_x^T G_x \chi + \chi^T G_x^T G_x \chi$$

$$+ \mu^T G_u^T G_u \mu + 2\mu^T G_u^T H_u u + u^T H_u^T H_u u)\mathrm{dt}$$

LQR in Frequency Domain

$$J = \int_0^\infty (x^T H_x^T H_x x + 2\, x^T H_x^T G_x \chi + \chi^T G_x^T G_x \chi$$

$$+ \mu^T G_u^T G_u \mu + 2\mu^T G_u^T H_u u + u^T H_u^T H_u u)\mathrm{d}t$$

$$= \int_0^\infty \left(z^T Q z + u^T R u + 2 z^T N u\right)\mathrm{d}t$$

LQR in Frequency Domain

$$J = \int_0^{\infty} \left(z^T Q z + u^T R u + 2 z^T N u \right) \mathrm{dt}$$

$$\left(\begin{array}{l} z = \begin{bmatrix} x \\ \chi \\ \mu \end{bmatrix} \quad Q = \begin{bmatrix} H_x^T H_x & H_x^T G_x & 0 \\ G_x^T H_x & G_x^T G_x & 0 \\ 0 & 0 & H_x^T H_x \end{bmatrix} \\ R = H_u^T H_u \quad N = \begin{bmatrix} 0 \\ 0 \\ G_u^T H_u \end{bmatrix} \end{array} \right)$$

LQR
in Frequency Domain

Filtering state is augmented and pluged into the LQR algorithm

$$\dot{z} = \bar{A}z + \bar{B}u$$

$$\left(\bar{A} = \begin{bmatrix} A & 0 & 0 \\ F_x & E_x & 0 \\ 0 & 0 & E_u \end{bmatrix} \quad \bar{B} = \begin{bmatrix} B \\ 0 \\ F_u \end{bmatrix} \right)$$

LQR in Frequency Domain

Control law

→ solve this ARE

$$P\bar{A} + \bar{A}^T P - (P\bar{B} + N)R^{-1}(P\bar{B} + N)^{\mathrm{T}} + Q = 0$$

» $u = R^{-1}(P\bar{B} + N)^{\mathrm{T}} z$

feedback gain: $K = R^{-1}(P\bar{B} + N)^{\mathrm{T}}$

State Estimator for Modal Damping

modal system: $\dot{q} = A_{\mathrm{m}} q + B_{\mathrm{m}} u \quad x = \phi q \quad y = Cx$

estimator: $\dot{\hat{q}} = A_{\mathrm{m}} \hat{q} + B_{\mathrm{m}} u - L_{\mathrm{m}}(C\hat{x} - y)$

estimator error: $\tilde{q} = \hat{q} - q$

$$\dot{\tilde{q}} = A_{\mathrm{m}} \tilde{x} - L_{\mathrm{m}} C_{\mathrm{m}} \tilde{q}$$

State Estimator for Modal Damping

$$\dot{\tilde{q}} = A_{\mathrm{m}}\tilde{x} - L_{\mathrm{m}}C_{\mathrm{m}}\tilde{q} \quad \ggg \quad \underline{\dot{\xi} = A_{\mathrm{m}}^T\xi + C_{\mathrm{m}}^T u}$$

sensor signal

transpose

fictious system which has the same dynamics

Filtering function and Filtered u

$$\dot{\mu} = E\mu + F^T u \qquad \bar{u} = G\mu + Hu$$

Augmented state space

$$\dot{z} = \bar{A}z + \bar{B}u \quad \left(\boldsymbol{z} = \begin{bmatrix} \xi \\ \mu \end{bmatrix} \quad \bar{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{A}_m^T & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{E} \end{bmatrix} \quad \bar{\boldsymbol{B}} = \begin{bmatrix} \boldsymbol{C}_m^T \\ \boldsymbol{F} \end{bmatrix} \right)$$

State Estimator for Modal Damping

Augmented feedback gain

$$\bar{L} = \begin{bmatrix} L_q^T & L_u^T \end{bmatrix} = \mathrm{lqr}(\bar{A}, \bar{B}, \boldsymbol{Q}, \boldsymbol{R}, \boldsymbol{N})$$

$$\left(\begin{array}{ll} \dot{z} = \bar{A}z - \bar{B}Lz & \boldsymbol{Q} = \begin{bmatrix} \mathbf{Q_m} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{G^T G} \end{bmatrix} \\ \boldsymbol{R} = \boldsymbol{H^T H} & \boldsymbol{N} = \begin{bmatrix} 0 \\ \boldsymbol{G_u^T H_u} \end{bmatrix} \end{array}\right)$$

mode reconstruction cost

$$\mathbf{Q_m} = \begin{bmatrix} \boldsymbol{H_x^T H_x} & \boldsymbol{H_x^T G_x} \\ \boldsymbol{G_x^T H_x} & \boldsymbol{G_x^T G_x} \end{bmatrix}$$